ANSWERS UNIT THREE

Exercise 1A PAGE 3

1 **a** $8i$ **b** $2\sqrt{2}i$ **c** $\sqrt{10}i$ **d** $3\sqrt{7}i$

2 **a** -5 **b** 3

3 **a** 12 **b** -5

4 **a** $\frac{3}{2}+\frac{\sqrt{3}}{2}i,\ \frac{3}{2}-\frac{\sqrt{3}}{2}i$ **b** $-2+\sqrt{3}i,\ -2-\sqrt{3}i$

c $\frac{1}{6}+\frac{\sqrt{11}}{6}i,\ \frac{1}{6}-\frac{\sqrt{11}}{6}i$ **d** $-0.8+0.4i, -0.8-0.4i$

5 $5+6i$ **6** -2 **7** $10-i$

8 $9+3i$ **9** $8-2i$ **10** $2i$

11 $7-6i$ **12** $17+6i$ **13** 2

14 -5 **15** $16+11i$ **16** $7+9i$

17 5 **18** $53i$ **19** $-0.8-1.4i$

20 $-\frac{5}{13}-\frac{12}{13}i$ **21** $\frac{7}{25}-\frac{26}{25}i$ **22** $-0.2+0.4i$

23 **a** $7+2i$ **b** $-3+4i$ **c** $-10+19i$

d $13+13i$ **e** $24-10i$ **f** $\frac{7}{26}+\frac{17}{26}i$

24 **a** $4+7i$ **b** 8

c 65 **d** $-\frac{33}{65}-\frac{56}{65}i$

25 $a=-34$ and $b=5$ **26** $a=10$ and $b=25$

27 **b** $p=-4, q=13$ **c** $d=-6, e=13$

28 **a** $(2, 3)$ **b** $(-3, 0)$

c $(4, 2)$ **d** $\left(-\frac{37}{169}, -\frac{55}{169}\right)$

29 $a=6$ and $b=0.5$ or $a=1$ and $b=3$

Exercise 1B PAGE 8

1 $p=-38$ **2** $a=2, b=1, c=5, d=8$

3 **a** -3 **b** -3

4 **a** 8 **b** 8

5 4 **6** -5 **7** $a=1, b=3$

8 **a** $f(-1)=-16, f(1)=0.$

b $x=1, x=1+2i, x=1-2i.$

c $x=0, x=1, x=1+2i, x=1-2i.$

9 **a** $f(-2)=0, f(2)=-36, f(-5)=1140, f(5)=0.$

b $x=-2, x=5, x=1+\sqrt{2}i, x=1-\sqrt{2}i.$

10 **a** $f(1)=2, f(0.5)=0.$ **b** $x=0.5, x=-i, x=i.$

11 $x=-1+i, x=-1-i, x=1-2i, x=1+2i.$

12 $x=1, x=\frac{1+3\sqrt{7}i}{4}, x=\frac{1-3\sqrt{7}i}{4}.$

13 $x=-1, x=0, x=\frac{3+\sqrt{3}i}{3}, x=\frac{3-\sqrt{3}i}{3}.$

Miscellaneous exercise one PAGE 9

1 **a** 58 **b** 26 **c** $12-5i$

d $-24-10i$ **e** $\frac{4}{5}-\frac{7}{5}i$ **f** $-\frac{1}{5}+\frac{2}{5}i$

2 **a** $-1+i$ **b** $8+31i$ **c** $3+4i$

d $-7-24i$ **e** $8-31i$ **f** $8-31i$

g $q=-4+4i$

3 $-4-4i$

4 18

5 **a** 6 **b** 8

6 0

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7 $a = -1, b = 2, c = -3, d = 6$

8 $p = q = -11$

9 **a** $6\mathbf{i} - 2\mathbf{j}$ **b** $\sqrt{2}(\mathbf{i} + 2\mathbf{j})$ **c** $d = \pm 3$
d 2 **e** 82°

10 $\mathbf{b} = -\mathbf{a}$ $\mathbf{c} = 2\mathbf{a}$ $\mathbf{d} = 0.5\mathbf{a}$
$\mathbf{e} = -0.5\mathbf{a}$ $\mathbf{f} = 1.5\mathbf{a}$ $\mathbf{g} = -1.5\mathbf{a}$

11 $\mathbf{r} = \mathbf{p} + \mathbf{q}$ $\mathbf{s} = 0.5\mathbf{p} + \mathbf{q}$ $\mathbf{t} = \mathbf{p} + 2\mathbf{q}$
$\mathbf{u} = -1.5\mathbf{p} - \mathbf{q}$

12 $x = 2, x = -4 + 2i, x = -4 - 2i.$

13 $a = \pm 2$ $b = 2$ $c = -7$
$d = 1$ $e = 5$ $f = -5$

Exercise 2A PAGE 15

1 **a** 5 **b** 13 **c** $\sqrt{13}$
d $\sqrt{13}$ **e** $\sqrt{26}$ **f** 5

2 **a** $\frac{\pi}{4}$ **b** $-\frac{\pi}{4}$ **c** $\frac{3\pi}{4}$
d $-\frac{3\pi}{4}$ **e** $\frac{2\pi}{3}$ **f** $-\frac{\pi}{3}$

3 $z_1 = 3\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

$z_2 = 3(\cos\pi + i\sin\pi)$

$z_3 = 4\left(\cos\left(-\frac{3\pi}{4}\right) + i\sin\left(-\frac{3\pi}{4}\right)\right)$

$z_4 = 2(\cos\pi + i\sin\pi)$

$z_5 = 6(\cos 1 + i\sin 1)$

$z_6 = 5\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

$z_7 = 8\left(\cos\left(-\frac{5\pi}{6}\right) + i\sin\left(-\frac{5\pi}{6}\right)\right)$

$z_8 = 5\left(\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)\right)$

$z_9 = 6(\cos 2 + i\sin 2)$

$z_{10} = 4(\cos\pi + i\sin\pi)$

$z_{11} = 5\left(\cos\left(-\frac{3\pi}{4}\right) + i\sin\left(-\frac{3\pi}{4}\right)\right)$

$z_{12} = 7\left(\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right)$

4 $z_{13} = 5\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$z_{14} = 5(\cos 2.2143 + i\sin 2.2143)$

$z_{15} = \sqrt{41}(\cos(-2.2455) + i\sin(-2.2455))$

$z_{16} = 5\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$

$z_{17} = 13(\cos 1.1760 + i\sin 1.1760)$

$z_{18} = 5\sqrt{2}(\cos 1.4289 + i\sin 1.4289)$

$z_{19} = 5\sqrt{2}(\cos(-1.4289) + i\sin(-1.4289))$

$z_{20} = 5\sqrt{2}(\cos 2.9997 + i\sin 2.9997)$

$z_{21} = 10\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

$z_{22} = 4\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

$z_{23} = 4(\cos 0 + i\sin 0)$

$z_{24} = 4(\cos\pi + i\sin\pi)$

$z_{25} = 3\left(\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)\right)$

$z_{26} = 3(\cos 0 + i\sin 0)$

5 $z_{27} = \sqrt{2} + \sqrt{2}i,$ $z_{28} = -2\sqrt{3} + 2i,$
$z_{29} = 2 - 2\sqrt{3}i,$ $z_{30} = -3 - 3\sqrt{3}i,$
$z_{31} = 5 + 0i,$ $z_{32} = 0 - i$

Exercise 2B PAGE 17

1 $z_1 = 3\,\text{cis}\,\frac{\pi}{3}$ $z_2 = 5\,\text{cis}\,\frac{2\pi}{3}$

$z_3 = 4\,\text{cis}\left(-\frac{5\pi}{6}\right)$ $z_4 = 5\,\text{cis}\left(-\frac{\pi}{2}\right)$

$z_5 = 4\,\text{cis}\,0$ $z_6 = 5\,\text{cis}\,\frac{\pi}{2}$

$z_7 = 5\,\text{cis}\,\frac{3\pi}{4}$ $z_8 = 3\,\text{cis}\left(-\frac{3\pi}{4}\right)$

2 $2\,\text{cis}\,\frac{\pi}{10}$ **3** $7\,\text{cis}\,\frac{5\pi}{8}$ **4** $9\,\text{cis}\,\frac{\pi}{6}$

5 $3\,\text{cis}\left(-\frac{\pi}{6}\right)$ **6** $5\,\text{cis}\left(-\frac{\pi}{2}\right)$ **7** $4\,\text{cis}\,\frac{2\pi}{3}$

8 $2\,\text{cis}\,\frac{\pi}{3}$ **9** $2\,\text{cis}\,\pi$ **10** $7i$

11 $-5i$ **12** -1 **13** 3

14 $5\sqrt{2} + 5\sqrt{2}i$ **15** $-2 + 2\sqrt{3}i$

16 $-2 - 2\sqrt{3}i$ **17** $-6 + 6\sqrt{3}i$

18 $25\,\text{cis}(1.8546)$ **19** $13\,\text{cis}(1.9656)$

20 $\sqrt{5}\,\text{cis}(1.1071)$ **21** $5\,\text{cis}\,\frac{\pi}{2}$

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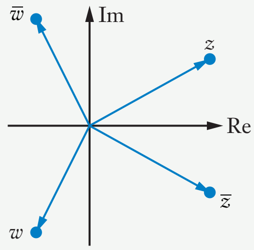
22 **a**



b $\overline{z} = r_1 \operatorname{cis}(-\alpha)$
$\overline{w} = r_2 \operatorname{cis}(-\beta)$

23 $2 \operatorname{cis}(-30°)$ **24** $7 \operatorname{cis}(-120°)$ **25** $4 \operatorname{cis}(-30°)$

26 $10 \operatorname{cis}(-160°)$ **27** $2 \operatorname{cis}\left(-\frac{\pi}{2}\right)$ **28** $5 \operatorname{cis}\frac{3\pi}{4}$

29 $5 \operatorname{cis}(-0.5)$ **30** $5 \operatorname{cis}\frac{\pi}{2}$

Exercise 2C PAGE 20

1 $16 + 11i$ **2** $-7 + 4i$ **3** $15 \operatorname{cis} 80°$

4 $9 \operatorname{cis}(-90°)$ **5** $9 \operatorname{cis}(-50°)$ **6** $10 \operatorname{cis}\frac{7\pi}{12}$

7 $8 \operatorname{cis}\left(-\frac{\pi}{2}\right)$ **8** $2(\cos 110° + i \sin 110°)$

9 $6(\cos(-40°) + i \sin(-40°))$ **10** $1.2 + 0.6i$

11 $1.2 + 0.6i$ **12** $4 \operatorname{cis} 20°$ **13** $5 \operatorname{cis}(-30°)$

14 $\operatorname{cis} 130°$ **15** $\operatorname{cis}\frac{\pi}{5}$ **16** $2 \operatorname{cis} \pi$

17 $2.5\left(\cos\frac{\pi}{4} + i \sin\frac{\pi}{4}\right)$

18 $0.4(\cos 0 + i \sin 0)$

19 $2 \operatorname{cis} 40°$ **20** $3 \operatorname{cis} 100°$ **21** $2 \operatorname{cis}(-90°)$

22 $2 \operatorname{cis} 120°$ **23** $\operatorname{cis} 160°$ **24** $2 \operatorname{cis}(-140°)$

25 $\operatorname{cis} 120°$ **26** $2 \operatorname{cis} 80°$ **27** $2 \operatorname{cis}(-100°)$

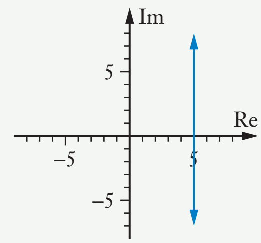
28 **a** $12 \operatorname{cis} 40°$ **b** $6 \operatorname{cis} 30°$
c $12 \operatorname{cis} 70°$ **d** $12 \operatorname{cis} 70°$
e $6 \operatorname{cis} 130°$ **f** $2 \operatorname{cis} 120°$
g $\frac{1}{3}\operatorname{cis}(-10°)$ **h** $\frac{1}{6}\operatorname{cis}(-40°)$

29 **a** $32 \operatorname{cis}\left(-\frac{7\pi}{12}\right)$ **b** $32 \operatorname{cis}\left(-\frac{7\pi}{12}\right)$
c $\frac{1}{2}\operatorname{cis}\left(\frac{\pi}{12}\right)$ **d** $2 \operatorname{cis}\left(-\frac{\pi}{12}\right)$
e $8 \operatorname{cis}\left(-\frac{2\pi}{3}\right)$ **f** $4 \operatorname{cis}\left(-\frac{3\pi}{4}\right)$
g $\frac{1}{8}\operatorname{cis}\left(-\frac{2\pi}{3}\right)$ **h** $\frac{1}{4}\operatorname{cis}\left(-\frac{\pi}{4}\right)$

Exercise 2D PAGE 25

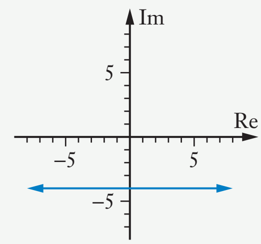
1 D **2** A **3** E
4 H **5** K **6** L
7 M **8** P

9



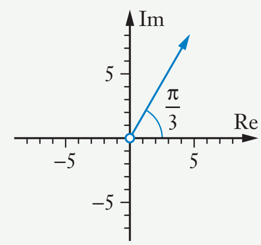
$x = 5$

10



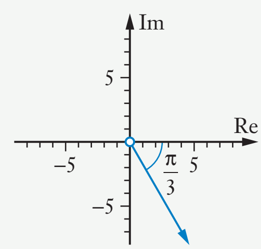
$y = -4$

11



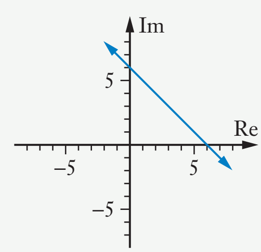
$y = \sqrt{3}x, x > 0$

12



$y = -\sqrt{3}x, x > 0$

13



$x + y = 6$

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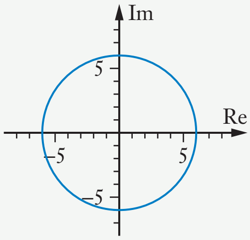
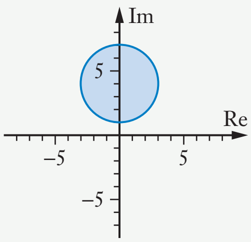
14



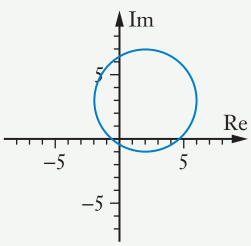
$x^2 + y^2 = 36$

15



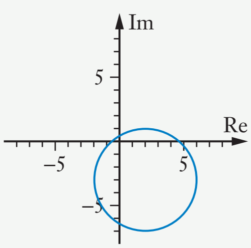
$x^2 + (y-4)^2 \le 9$

16



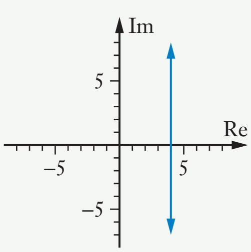
$(x-2)^2 + (y-3)^2 = 16$

17



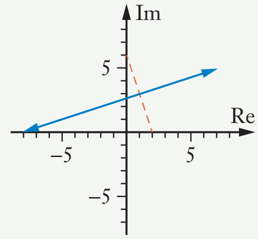
$(x-2)^2 + (y+3)^2 = 16$

18



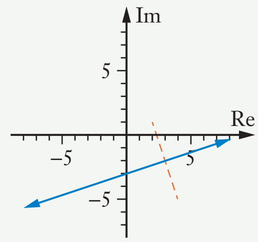
$x = 4$

19



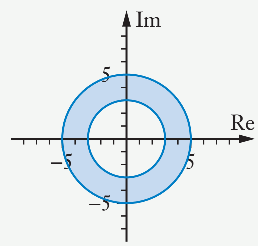
$3y = x + 8$

20



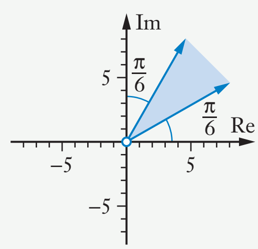
$3y = x - 9$

21



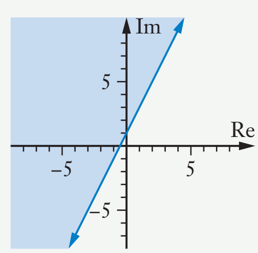
$9 \le x^2 + y^2 \le 25$

22



$\frac{1}{\sqrt{3}}x \le y \le x\sqrt{3}, x > 0$

23



$y \ge 2x + 1$

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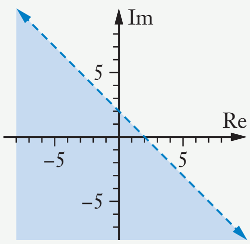
24



$y + x < 2$

(Note the use of the dashed line in question 24 because the question involved < rather than ≤.)

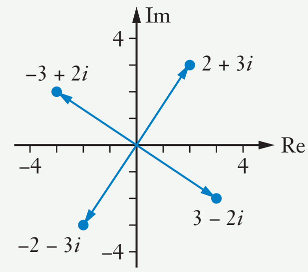
25 **a** 1 **b** 5 **c** $3\sqrt{2} - 2$
d $3\sqrt{2} + 2$ **e** $3\sqrt{2} + 2$

26 **a** 1 **b** 6 **c** 7
d 3 **e** 0.23 rads **f** 1.06 rads

27 Points $z = x + iy$ satisfy the equation $(x - 6)^2 + (y + 5)^2 = 20$, i.e. a circle, centre (6, −5), radius $2\sqrt{5}$.

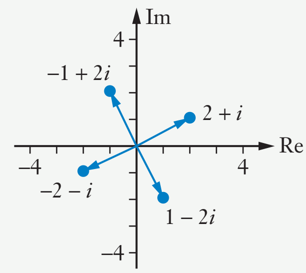
28 Points $z = x + iy$ satisfy the equation $(x - 1)^2 + (y + 4)^2 = 18$, i.e. a circle, centre (1, −4), radius $3\sqrt{2}$.

6



7 **a** $3 + 4i$ **b** $-7 + 24i$

c and **d**



8 $k = 32 \operatorname{cis} 100°$. The other solutions are $2 \operatorname{cis} 92°$, $2 \operatorname{cis} 164°$, $2 \operatorname{cis}(-52°)$, $2 \operatorname{cis}(-124°)$

9 $-4 + 2i, -2 - 4i, 4 - 2i$

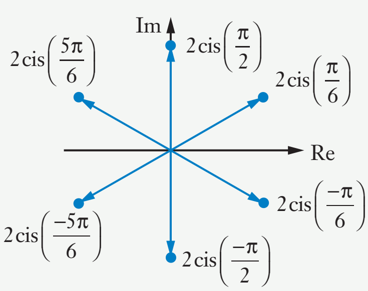
Exercise 2E PAGE 30

1 $1 \operatorname{cis} 0$ (i.e. 1), $1 \operatorname{cis}\left(\frac{\pi}{3}\right)$, $1 \operatorname{cis}\left(\frac{2\pi}{3}\right)$, $1 \operatorname{cis} \pi$, $1 \operatorname{cis}\left(-\frac{\pi}{3}\right)$, $1 \operatorname{cis}\left(-\frac{2\pi}{3}\right)$.

2 $1 \operatorname{cis} 0$, $1 \operatorname{cis} 45°$, $1 \operatorname{cis} 90°$, $1 \operatorname{cis} 135°$, $1 \operatorname{cis} 180°$, $1 \operatorname{cis}(-135°)$, $1 \operatorname{cis}(-90°)$, $1 \operatorname{cis}(-45°)$.

3 $1 \operatorname{cis} 0$, $1 \operatorname{cis}\left(\frac{2\pi}{7}\right)$, $1 \operatorname{cis}\left(\frac{4\pi}{7}\right)$, $1 \operatorname{cis}\left(\frac{6\pi}{7}\right)$, $1 \operatorname{cis}\left(-\frac{2\pi}{7}\right)$, $1 \operatorname{cis}\left(-\frac{4\pi}{7}\right)$, $1 \operatorname{cis}\left(-\frac{6\pi}{7}\right)$.

4



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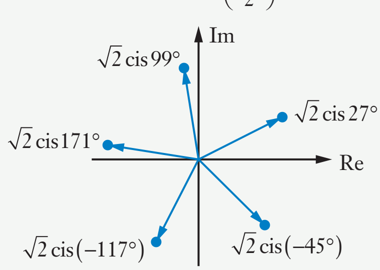

Exercise 2F PAGE 34

2 $\cos\left(\frac{2\pi}{3}\right) + i \sin\left(\frac{2\pi}{3}\right)$

3 $32 \operatorname{cis}\left(\frac{5\pi}{6}\right)$

4 $243\left(\cos\left(-\frac{\pi}{3}\right) + i \sin\left(-\frac{\pi}{3}\right)\right)$

5 $\cos 2\theta = \cos^2\theta - \sin^2\theta$, $\sin 2\theta = 2\sin\theta\cos\theta$

6 $\cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta$,
$\sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta$,
$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

7 $\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$,
$\sin 5\theta = 5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta$

8 $8 \operatorname{cis}\left(-\frac{\pi}{2}\right)$ **9** $32 \operatorname{cis}\left(\frac{5\pi}{6}\right)$ **10** $6^4 \operatorname{cis}\left(\frac{2\pi}{3}\right)$

11 $2 \operatorname{cis}\left(-\frac{\pi}{9}\right)$, $2 \operatorname{cis}\left(\frac{5\pi}{9}\right)$, $2 \operatorname{cis}\left(-\frac{7\pi}{9}\right)$

12 $2 \operatorname{cis}\left(\frac{\pi}{8}\right)$, $2 \operatorname{cis}\left(\frac{5\pi}{8}\right)$, $2 \operatorname{cis}\left(-\frac{7\pi}{8}\right)$, $2 \operatorname{cis}\left(-\frac{3\pi}{8}\right)$

13 $2 \operatorname{cis}\left(\frac{3\pi}{16}\right)$, $2 \operatorname{cis}\left(\frac{11\pi}{16}\right)$, $2 \operatorname{cis}\left(-\frac{13\pi}{16}\right)$, $2 \operatorname{cis}\left(-\frac{5\pi}{16}\right)$

14 $\sqrt{2} \operatorname{cis}\left(\frac{\pi}{4}\right)$, $\sqrt{2} \operatorname{cis}\left(\frac{3\pi}{4}\right)$, $\sqrt{2} \operatorname{cis}\left(-\frac{3\pi}{4}\right)$, $\sqrt{2} \operatorname{cis}\left(-\frac{\pi}{4}\right)$

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15 $z_1 = \sqrt{2}\,\text{cis}\left(\frac{\pi}{3}\right)$, $z_2 = \sqrt{2}\,\text{cis}\left(\frac{\pi}{6}\right)$, $\sqrt{2}$

16 **a** $r\,\text{cis}(\pi - \theta)$ **b** $\frac{1}{r}\text{cis}(-\theta)$

c $\frac{1}{r}\text{cis}(\pi - \theta)$ **d** $\frac{1}{r^2}\text{cis}(\pi - 2\theta)$

Miscellaneous exercise two PAGE 36

1 **a** $5 - i$ **b** $1 - 7i$ **c** $18 + i$

d $-7 - 24i$ **e** $-\frac{6}{13} - \frac{17}{13}i$ **f** $-\frac{6}{25} + \frac{17}{25}i$

2 **a** **c** **b** $\frac{1}{4}\mathbf{c}$ **c** $\frac{3}{4}\mathbf{c}$

d $\frac{5}{4}\mathbf{c}$ **e** $\mathbf{a} + \mathbf{c}$ **f** $\mathbf{a} + \frac{1}{4}\mathbf{c}$

g $\mathbf{a} + \frac{1}{2}\mathbf{c}$ **h** $\mathbf{a} + \frac{3}{2}\mathbf{c}$

3 **a** $6\,\text{cis}\left(-\frac{2\pi}{3}\right)$ **b** $-4\sqrt{3} - 4i$

4 **a** $(0, 2)$ **b** $(-5, 0)$ **c** $(-2\sqrt{2}, -2\sqrt{2})$

5 $\sqrt{2}\,\text{cis}\left(\frac{\pi}{4}\right)$, $\sqrt{2}\,\text{cis}\left(\frac{3\pi}{4}\right)$, $2\,\text{cis}\,\pi$, $\text{cis}\left(-\frac{\pi}{2}\right)$

7 **a** $f(-3) = -348$, $f(3) = 0$

b $x = 3, x = \frac{3}{4} + \frac{\sqrt{7}}{4}i, x = \frac{3}{4} - \frac{\sqrt{7}}{4}i$

Exercise 3A PAGE 46

1 **a** $\{-1, 1, 3, 5, 7\}$ **b** $\{-2, 0, 2, 4, 6\}$

c $\{-9, -5, -1, 3, 7\}$

2 **a** $\{9, 16, 25\}$ **b** $\{3, 52, 679\}$

c $\{729, 4096, 15\,625\}$

3 **a** Domain $\mathbb{R}$ Range $\mathbb{R}$

b Domain $\mathbb{R}$ Range $\mathbb{R}$

c Domain $\mathbb{R}$ Range $\mathbb{R}$

d Domain $\mathbb{R}$ Range $\{y \in \mathbb{R}: y = 10\}$

e Domain $\mathbb{R}$ Range $\{y \in \mathbb{R}: y \geq -25\}$

f Domain $\{x \in \mathbb{R}: x \neq 5\}$ Range $\{y \in \mathbb{R}: y \neq 1\}$

4 **a** $gf(x)$ **b** $hf(x)$

c $fg(x)$ **d** $fh(x)$

e $gh(x)$ **f** $hg(x)$

g $ff(x)$ **h** $hh(x)$

i $fff(x)$

5 **a** $4x - 9$ **b** $16x + 5$

c $x^4 + 2x^2 + 2$ **d** $8x - 1$

e $8x - 11$ **f** $2x^2 - 1$

g $4x^2 - 12x + 10$ **h** $4x^2 + 5$

i $16x^2 + 8x + 2$

6 **a** $4x + 15$ **b** $9x + 4$ **c** $\frac{3x + 2}{x + 2}$

d $6x + 7$ **e** $6x + 16$ **f** $7 + \frac{4}{x}$

g $\frac{2x + 7}{2x + 5}$ **h** $4 + \frac{6}{x}$ **i** $\frac{3(x + 1)}{3x + 1}$

7 $\{x \in \mathbb{R}: x \geq 4\}$ **8** $\{x \in \mathbb{R}: x \leq 4\}$

9 $\{x \in \mathbb{R}: -2 \leq x \leq 2\}$ **10** $\{x \in \mathbb{R}: -4 \leq x \leq 4\}$

11 $\{x \in \mathbb{R}: x \geq 2\}$ **12** $\{x \in \mathbb{R}: x \geq 3\}$

13 **a** 12 **b** 12 **c** 0.5

d 4 **e** 0.25

f Domain $\mathbb{R}$, Range $\{y \in \mathbb{R}: y \geq 3\}$

g Domain $\{x \in \mathbb{R}: x \neq 0\}$, Range $\{y \in \mathbb{R}: y \neq 0\}$

h Domain $\mathbb{R}$, Range $\{y \in \mathbb{R}: 0 < y \leq \frac{1}{3}\}$

i Domain $\{x \in \mathbb{R}: x \neq 0\}$, Range $\{y \in \mathbb{R}: y > 3\}$

14 **a** 0 **b** 0 **c** 2

d 21 **e** 3

f Domain $\mathbb{R}$, Range $\{y \in \mathbb{R}: y \leq 25\}$

g Domain $\{x \in \mathbb{R}: x \geq 0\}$, Range $\{y \in \mathbb{R}: y \geq 0\}$

h Domain $\{x \in \mathbb{R}: -5 \leq x \leq 5\}$, Range $\{y \in \mathbb{R}: 0 \leq y \leq 5\}$

i Domain $\{x \in \mathbb{R}: x \geq 0\}$, Range $\{y \in \mathbb{R}: y \leq 25\}$

15 **a** Domain $\{x \in \mathbb{R}: x \neq 1\}$, Range $\{y \in \mathbb{R}: y \neq 0\}$

b Domain $\{x \in \mathbb{R}: x \neq 3\}$, Range $\{y \in \mathbb{R}: y \neq 2\}$

16 **a** Domain $\{x \in \mathbb{R}: x \geq 0\}$, Range $\{y \in \mathbb{R}: y \geq -1\}$

b Domain $\{x \in \mathbb{R}: x \geq 0.5\}$, Range $\{y \in \mathbb{R}: y \geq 0\}$

17 **a** Domain $\{x \in \mathbb{R}: x \neq 0\}$, Range $\{y \in \mathbb{R}: y > 0\}$

b Domain $\{x \in \mathbb{R}: x > 0\}$, Range $\{y \in \mathbb{R}: y > 0\}$

20 **a** Domain $\{x \in \mathbb{R}: x \neq \pm 3\}$,

Range $\{y \in \mathbb{R}: y \leq -\frac{1}{9}\} \cup \{y \in \mathbb{R}: y > 0\}$

where $\cup$ means the two sets are *united* to give the complete range.

b Domain $\{x \in \mathbb{R}: x \neq 0\}$, Range $\{y \in \mathbb{R}: y > -9\}$

Exercise 3B PAGE 54

1 a, b, c, g, h

2 $x + 2$, Domain $\mathbb{R}$, Range $\mathbb{R}$

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3 $\dfrac{x+5}{2}$, Domain $\mathbb{R}$, Range $\mathbb{R}$

4 $\dfrac{x-2}{5}$, Domain $\mathbb{R}$, Range $\mathbb{R}$

5 $\dfrac{1}{x}+4$, Domain $x \neq 0$, Range $y \neq 4$

6 $\dfrac{1}{x}-3$, Domain $x \neq 0$, Range $y \neq -3$

7 $\dfrac{1+5x}{2x}$, Domain $x \neq 0$, Range $y \neq 2.5$

8 $\dfrac{1}{x-1}-2$, Domain $x \neq 1$, Range $y \neq -2$

9 $\dfrac{1}{3-x}+1$, Domain $x \neq 3$, Range $y \neq 1$

10 $\dfrac{1}{x-4}+\dfrac{1}{2}$, Domain $x \neq 4$, Range $y \neq 0.5$

11 x^2, Domain $x \geq 0$, Range $y \geq 0$

12 x^2-1, Domain $x \geq 0$, Range $y \geq -1$

13 $\dfrac{x^2+3}{2}$, Domain $x \geq 0$, Range $y \geq 1.5$

14 $\dfrac{x-5}{2}$

15 $\dfrac{x-1}{3}$

16 $\dfrac{2}{x-1}$

17 x

18 x

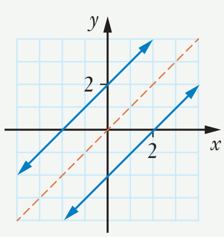
19 $\dfrac{4}{x-1}+5$

20 $\dfrac{x-7}{6}$

21 $\dfrac{x-7}{6}$

22 $\dfrac{2x+13}{3}$

23 **a** A function one-to-one



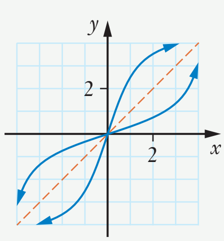
b Not a function

c A function, not one-to-one

d Not a function

e Not a function

f A function, one-to-one



24 For $f(x)$ restricted to $x \geq 0$ then $f^{-1}(x) = \sqrt{x-3}$, domain $x \geq 3$ and range $y \geq 0$.
(Or restrict $f(x)$ to $x \leq 0$ then $f^{-1}(x) = -\sqrt{x-3}$, domain $x \geq 3$ and range $y \leq 0$.)

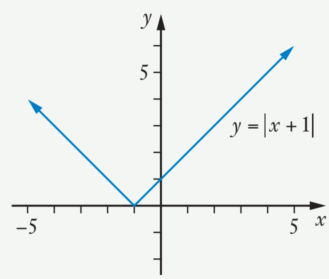
25 For $f(x)$ restricted to $x \geq -3$ then $f^{-1}(x) = -3+\sqrt{x}$, domain $x \geq 0$ and range $y \geq -3$
(Or restrict $f(x)$ to $x \leq -3$ then $f^{-1}(x) = -3-\sqrt{x}$, domain $x \geq 0$ and range $y \leq -3$)

26 For $f(x)$ restricted to $x \geq 3$ then $f^{-1}(x) = 3+\sqrt{x-2}$, domain $x \geq 2$ and range $y \geq 3$
(Or restrict $f(x)$ to $x \leq 3$ then $f^{-1}(x) = 3-\sqrt{x-2}$, domain $x \geq 2$ and range $y \leq 3$)

27 For $f(x)$ restricted to $0 \leq x \leq 2$ then $f^{-1}(x) = \sqrt{4-x^2}$, domain $0 \leq x \leq 2$ and range $0 \leq y \leq 2$
(Or restrict $f(x)$ to $-2 \leq x \leq 0$ then $f^{-1}(x) = -\sqrt{4-x^2}$, domain $0 \leq x \leq 2$ and range $-2 \leq y \leq 0$.)

Exercise 3C PAGE 64

1



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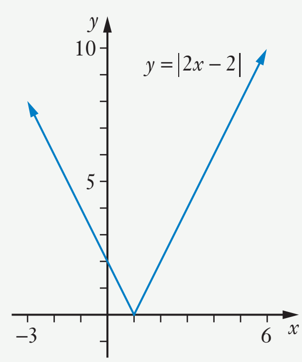

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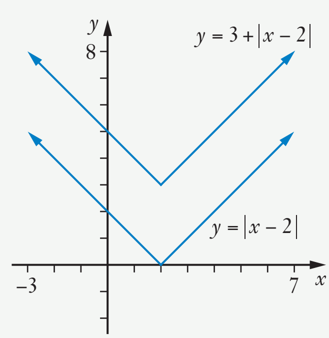

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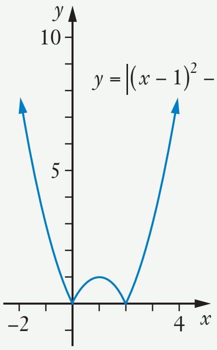

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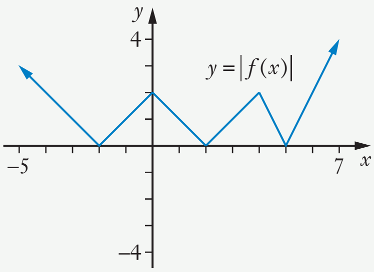

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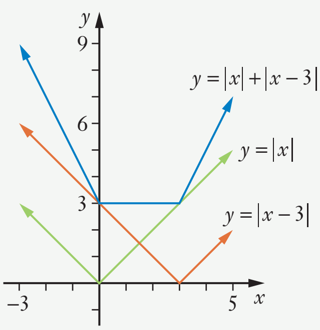
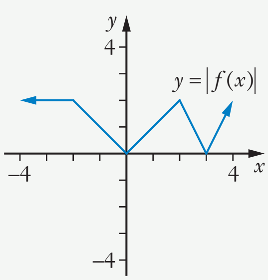


7 a



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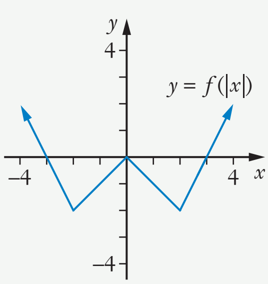
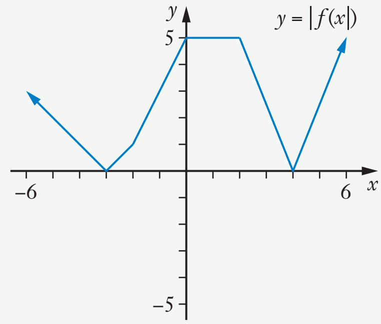


8 a



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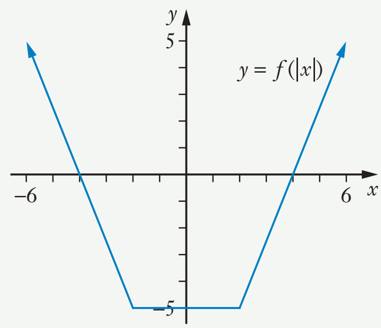
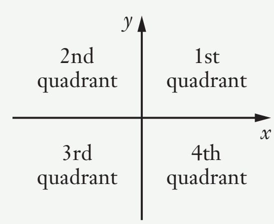


9 In the 1st and 4th quadrants (see the diagram below) the graph of $y = g(|x|)$ will be the same as that of $y = g(x)$.

However, in the 2nd and 3rd quadrants the graph of $y = g(|x|)$ will be those parts of $y = g(x)$ that lie in the 1st and 4th quadrants, reflected in the y-axis.



10 a The function $g(x) = (x + 1)^2$ has domain $\mathbb{R}$ and range $\{y \in \mathbb{R}: y \geq 0\}$.

The function $f(x) = 2 + \sqrt{x}$ has domain $\{x \in \mathbb{R}: x \geq 0\}$ and range $\{y \in \mathbb{R}: y \geq 2\}$.

Thus $g(x)$ is defined for all real x and the output from $g(x)$ consists of numbers that are all within the domain of $f(x)$. Thus $f[g(x)]$ is defined for all real x.

$$\mathbb{R} \to \boxed{g(x) = (x + 1)^2} \to y \in \mathbb{R}: y \geq 0$$

$$\to \boxed{f(x) = 2 + \sqrt{x}} \to y \in \mathbb{R}: y \geq 2$$

Thus $f[g(x)]$ has domain $\mathbb{R}$ and range $\{y \in \mathbb{R}: y \geq 2\}$.

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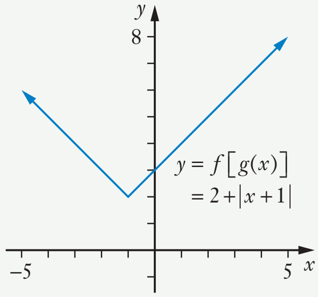
b $f[g(x)] = 2 + \sqrt{(x+1)^2}$
$= 2 + |x+1|$

c



11 a The function $g(x) = (x-2)^2$ has domain $\mathbb{R}$ and range $\{y \in \mathbb{R}: y \geq 0\}$.
The function $f(x) = 1 - \sqrt{x}$ has domain $\{x \in \mathbb{R}: x \geq 0\}$ and range $\{y \in \mathbb{R}: y \leq 1\}$.
Thus $g(x)$ is defined for all real x and the output from $g(x)$ consists of numbers that are all within the domain of $f(x)$. Thus $f[g(x)]$ is defined for all real x.

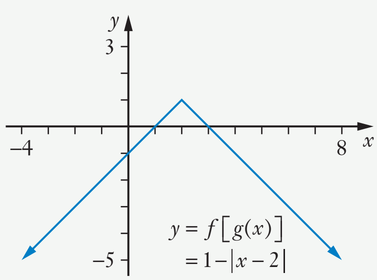
$\mathbb{R} \rightarrow \boxed{g(x) = (x-2)^2} \rightarrow y \in \mathbb{R}: y \geq 0$

$\rightarrow \boxed{f(x) = 1 - \sqrt{x}} \rightarrow y \in \mathbb{R}: y \leq 1$

Thus $f[g(x)]$ has domain $\mathbb{R}$ and range $\{y \in \mathbb{R}: y \leq 1\}$.

b $f[g(x)] = 1 - \sqrt{(x-2)^2} = 1 - |x-2|$

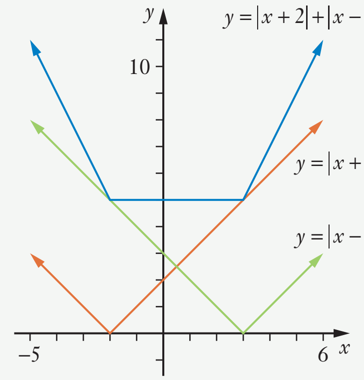
c



12 a $x = 3, x = 7$ **b** $x = -2, x = 6$
c $x = 4, x = 8$

13 Graph not shown here.
a $x = -4, x = 1$ **b** $x = -6, x = 2$
c $x = -4, x = 0$ **d** $x = -3, x = -1$

14 a, b and **c.**



d $-4 \leq x \leq 5$

15 $x = -7, x = -5$
16 No solutions
17 $x = 8$
18 $x = 3, x = 19$
19 $x = 1$
20 $x = -5.5, x = 1.5$
21 $-5 \leq x \leq 3$
22 $x \geq 8$
23 $x \leq -1$
24 $\mathbb{R}$
25 $x \leq 3$
26 $\mathbb{R}$
27 $>, a = 11, b = -8$
28 $\leq, a = 7$
29 $<, a = 1$
30 $a = -0.5, b = 8, c = 3$

Exercise 3D PAGE 75

1 $x = 0$
2 $x = 1$
3 $x = 3$ and $x = 0.5$
4 $x = 3$
5 Cannot have $y = 0$.
6 Cannot have $y = 2$
7 Cannot have $y = 0$.
8 Cannot have $y = 1$

9 As $x \rightarrow +\infty$, then $y \rightarrow 0^+$
$x \rightarrow -\infty$, then $y \rightarrow 0^-$

10 As $x \rightarrow +\infty$, then $y \rightarrow 1^+$
$x \rightarrow -\infty$, then $y \rightarrow 1^-$

11 As $x \rightarrow +\infty$, then $y \rightarrow 5^+$
$x \rightarrow -\infty$, then $y \rightarrow 5^-$

12 As $x \rightarrow +\infty$, then $y \rightarrow 3^+$
$x \rightarrow -\infty$, then $y \rightarrow 3^-$

13 As $x \rightarrow 3^+$, then $y \rightarrow +\infty$
$x \rightarrow 3^-$, then $y \rightarrow -\infty$

14 As $x \rightarrow 1^+$, then $y \rightarrow -\infty$
$x \rightarrow 1^-$, then $y \rightarrow +\infty$

15 As $x \rightarrow 0^+$, then $y \rightarrow +\infty$
$x \rightarrow 0^-$, then $y \rightarrow +\infty$

16 a $y = \dfrac{1}{(x-3)^2}$ **b** $y = \dfrac{1}{(x+3)(x-3)}$

c $y = \dfrac{1}{x-3}$

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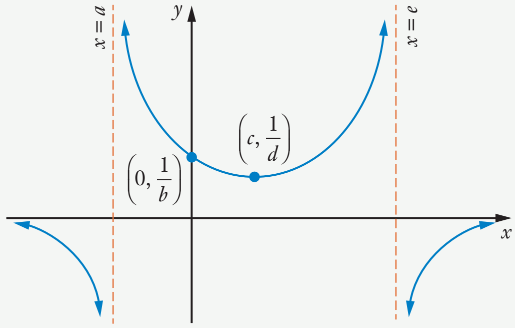
18



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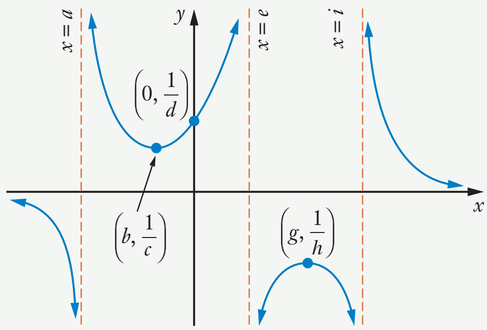

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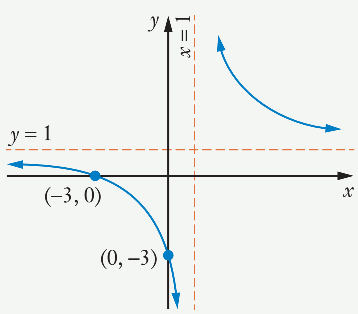

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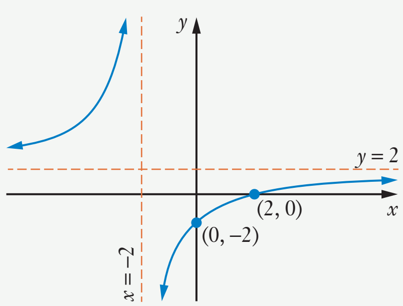

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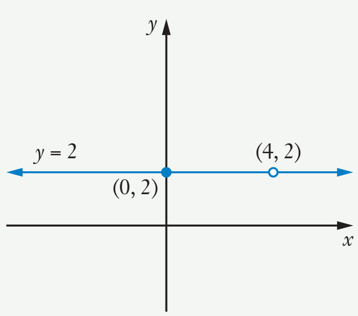

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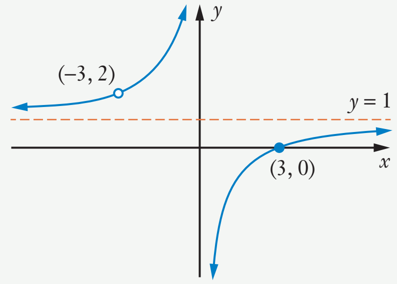

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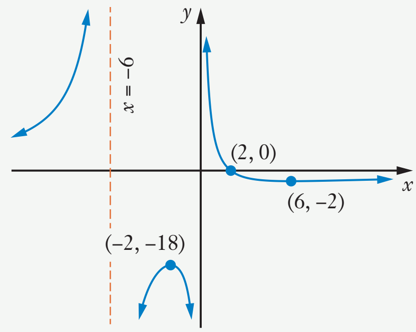

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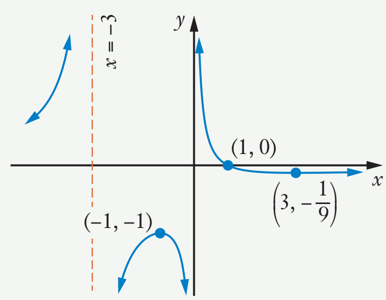

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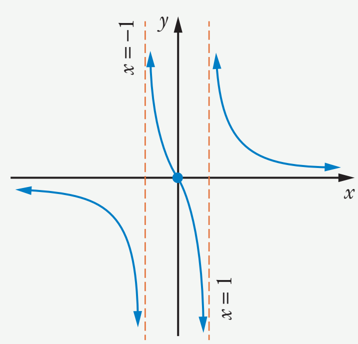

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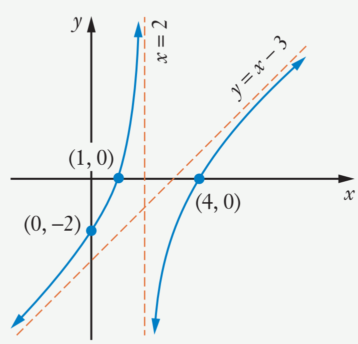

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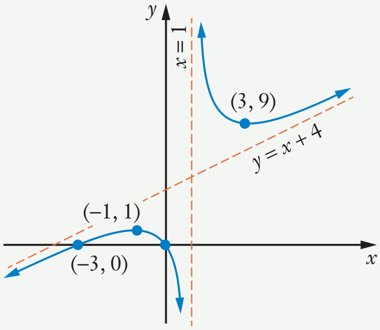

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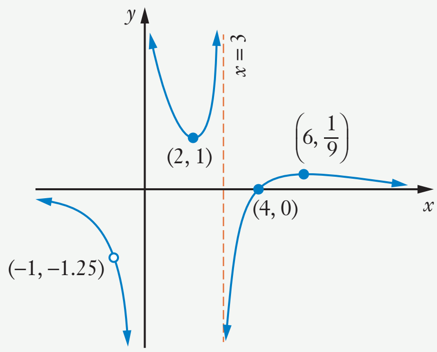

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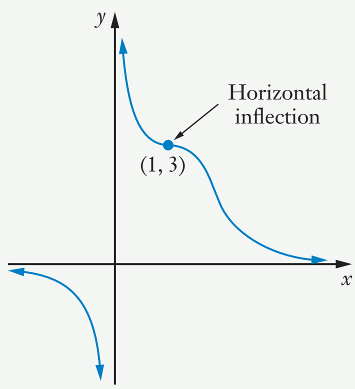

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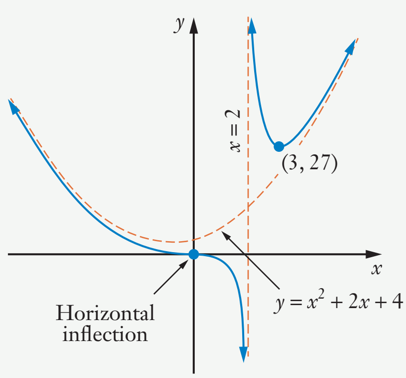
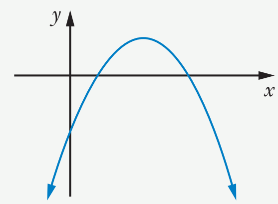


Miscellaneous exercise three PAGE 79

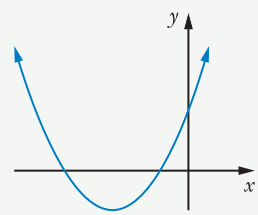
1 $x = -1, x = -3 - 2i, x = -3 + 2i$

2 $a = -3, b = 1$, C has coordinates (–1, –0.25).

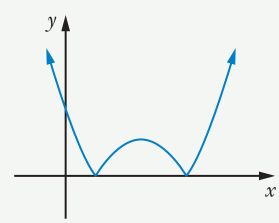
3 $x = -f(x)$



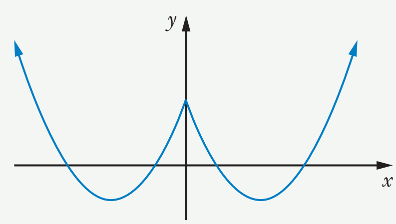
$x = f(-x)$



$y = |f(x)|$



$y = f(|x|)$



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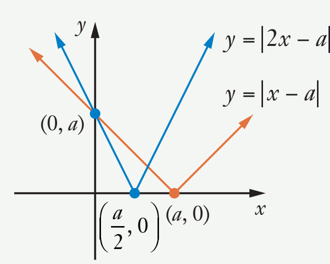

$0 \le x \le \frac{2}{3}a$

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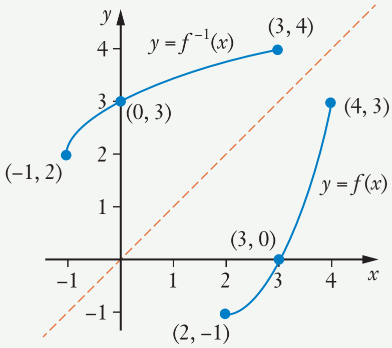
5 **a** $f(x)$: Domain $2 \le x \le 4$, Range $-1 \le y \le 3$

b $f^{-1}(x)$: Domain $-1 \le x \le 3$, Range $2 \le y \le 4$

c Sketch of $f(x)$ and $f^{-1}(x)$ shown below.



d $f^{-1}(x) = 2 + \sqrt{x+1}$ for $-1 \le x \le 3$

6 **a** $p = q = 0$ **b** $p = 3, q = 0$

c $p = -2, q = 1$ **d** $p = 3, q = -2$

e $p = 4, q = -1$ **f** $p = -3, q = 1$

7 **a** $3\mathbf{a} - 3\mathbf{b}$ **b** $-3\mathbf{a} + 2\mathbf{b}$

c $\frac{3}{2}\mathbf{a} - 2\mathbf{b}$ **d** $\frac{1}{2}\mathbf{a} - 7\mathbf{b}$

8 **a** $1 - \sqrt{3}i$ **b** $2\,\text{cis}\left(-\frac{\pi}{3}\right)$

9 $\frac{\sqrt{3}}{8} + i\frac{1}{8}$ **10** $p = iz, q = -z, w = -iz$

11 **a** $2\,\text{cis}\frac{5\pi}{12}$ **b** $2\,\text{cis}\frac{\pi}{12}$ **c** $1\,\text{cis}\frac{\pi}{3}$

d $8\,\text{cis}\frac{3\pi}{4}$ **e** $1\,\text{cis}\left(-\frac{\pi}{2}\right)$ **f** $512\,\text{cis}\frac{\pi}{4}$

12 $2\,\text{cis}\frac{5\pi}{6}$, 2^{12} $(= 4096)$

Exercise 4A PAGE 86

1 $\mathbf{r}_A(t) = [(5 + 10t)\mathbf{i} + (4 - t)\mathbf{j}]$ km,
$\mathbf{r}_B(t) = [(6 + 2t)\mathbf{i} + (8t - 8)\mathbf{j}]$ km
$\mathbf{r}_C(t) = [(2 - 4t)\mathbf{i} + (3 + 3t)\mathbf{j}]$ km,
$\mathbf{r}_D(t) = [(19 + 10t)\mathbf{i} + (6t - 4)\mathbf{j}]$ km
$\mathbf{r}_E(t) = [(20 - 4t)\mathbf{i} + (4 + 3t)\mathbf{j}]$ km, $t \ge 1$,
$\mathbf{r}_F(t) = [(12t - 4)\mathbf{i} + (7 - 8t)\mathbf{j}]$ km, $t \ge 0.5$

2 **a** $(10\mathbf{i} + 14\mathbf{j})$ km **b** $(13\mathbf{i} + 18\mathbf{j})$ km

c $(19\mathbf{i} + 26\mathbf{j})$ km **d** 5 km/h

e $\sqrt{29}$ km

3 $(7\mathbf{i} + 24\mathbf{j})$ km **a** 25 km **b** 13 km

4 **a** $\sqrt{185}$ km **b** $\sqrt{65}$ km **c** $\sqrt{13}$ km

5 **a** 13 km **b** 17 km

6 **a** $\mathbf{r}_A(t) = (28 - 8t)\mathbf{i} + (4t - 5)\mathbf{j}$,
$\mathbf{r}_B(t) = 6t\mathbf{i} + (24 + 2t)\mathbf{j}$

b At 10 a.m. and again at 10:30 a.m.

7 Collision. 1 p.m., $(47\mathbf{i} + 21\mathbf{j})$ km.

8 No collision.

9 Collision. 3 p.m., $(3\mathbf{i} + 3\mathbf{j}$ km.

10 Collision. 2 p.m., $(12\mathbf{i} + 17\mathbf{j})$ km.

11 No collision.

12 **a** Q and R, 10:30 a.m., $(37\mathbf{i} + 5\mathbf{j})$ km

b 17 km.

Exercise 4B PAGE 92

1 $\mathbf{r} = (2 + 5\lambda)\mathbf{i} + (3 - \lambda)\mathbf{j}$ **2** $\mathbf{r} = (3 + \lambda)\mathbf{i} + (\lambda - 2)\mathbf{j}$

3 $\mathbf{r} = 5\mathbf{i} + (3 - 2\lambda)\mathbf{j}$ **4** $\mathbf{r} = 3\lambda\mathbf{i} + (5 - 10\lambda)\mathbf{j}$

5 $\mathbf{r} = \begin{pmatrix} 2 + \lambda \\ -3 + 4\lambda \end{pmatrix}$ **6** $\mathbf{r} = \begin{pmatrix} 5\lambda \\ 5 \end{pmatrix}$

7 $\mathbf{r} = (5\mathbf{i} + 3\mathbf{j}) + \lambda(-3\mathbf{i} - 4\mathbf{j})$
i.e. $\mathbf{r} = (5 - 3\lambda)\mathbf{i} + (3 - 4\lambda)\mathbf{j}$

8 $\mathbf{r} = (6\mathbf{i} + 7\mathbf{j}) + \lambda(-11\mathbf{i} - 5\mathbf{j})$
i.e. $\mathbf{r} = (6 - 11\lambda)\mathbf{i} + (7 - 5\lambda)\mathbf{j}$

9 $\mathbf{r} = \begin{pmatrix} -6 \\ 3 \end{pmatrix} + \lambda\begin{pmatrix} 8 \\ 1 \end{pmatrix}$ i.e. $\mathbf{r} = \begin{pmatrix} -6 + 8\lambda \\ 3 + \lambda \end{pmatrix}$

10 $\mathbf{r} = \begin{pmatrix} 1 \\ -3 \end{pmatrix} + \lambda\begin{pmatrix} -4 \\ 4 \end{pmatrix}$ i.e. $\mathbf{r} = \begin{pmatrix} 1 - 4\lambda \\ -3 + 4\lambda \end{pmatrix}$

11 $\mathbf{r} = \begin{pmatrix} 1 \\ 4 \end{pmatrix} + \lambda\begin{pmatrix} 2 \\ -5 \end{pmatrix}$ i.e. $\mathbf{r} = \begin{pmatrix} 1 + 2\lambda \\ 4 - 5\lambda \end{pmatrix}$

12 $\mathbf{r} = \begin{pmatrix} 5 \\ 0 \end{pmatrix} + \lambda\begin{pmatrix} -6 \\ -4 \end{pmatrix}$ i.e. $\mathbf{r} = \begin{pmatrix} 5 - 6\lambda \\ -4\lambda \end{pmatrix}$

13 **a** $2\mathbf{i} - 8\mathbf{j}$ **b** $\sqrt{17}$ units **c** $2:1$

14 **a** $\mathbf{r} = (5 + 7\lambda)\mathbf{i} + (2\lambda - 1)\mathbf{j}$

b $x = 5 + 7\lambda, y = 2\lambda - 1$

c $7y = 2x - 17$

15 **a** $\mathbf{r} = \begin{pmatrix} 2 - 3\lambda \\ -1 + 4\lambda \end{pmatrix}$ **b** $x = 2 - 3\lambda, y = -1 + 4\lambda$

c $4x + 3y = 5$

16 **a** $\mathbf{r} = \begin{pmatrix} 7\lambda \\ 3 - 8\lambda \end{pmatrix}$ **b** $x = 7\lambda, y = 3 - 8\lambda$

c $8x + 7y = 21$

17 **a** $\mathbf{r} = \begin{pmatrix} 2 - 3\lambda \\ -5 + 2\lambda \end{pmatrix}$ **b** $2x + 3y + 11 = 0$

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18 a $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$ **b** $\begin{pmatrix} 3 \\ -9 \end{pmatrix}$ **c** $3\sqrt{10}$

d $3:1$ **e** $3:-1$ **f** $3:1$

19 $\mathbf{r} = (7 - 2\lambda)\mathbf{i} + (6\lambda - 2)\mathbf{j}$. B and C lie on the line, D and E do not.

20 $\mathbf{r} = \begin{pmatrix} 4-\lambda \\ -9+2\lambda \end{pmatrix}$. H and I lie on the line, G does not.

21 $a = -9, b = 21, c = -17, d = -12, e = 11, f = 15$

22 $\mathbf{r} = (5 + \lambda)\mathbf{i} - (6 + \lambda)\mathbf{j}$

23 $\mathbf{r} = \begin{pmatrix} 6+3\lambda \\ 5-4\lambda \end{pmatrix}$ **24** $5x + 3y = 46$

25 $6\mathbf{i}, 12\mathbf{j}$ **26** $-3\mathbf{i}, -7$

27 $\mathbf{r} = (2 + 3\lambda)\mathbf{i} + (3 - 7\lambda)\mathbf{j}, \frac{2}{7}, \frac{37}{3}$

28 8, 19 **29** $7, \frac{4}{3}$

30 Set (1). The other two both give the Cartesian equation $2y = x + 4$ but (1) gives $2y = x + 5$.

32 $\mathbf{r} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} + \mu\begin{pmatrix} -2 \\ 3 \end{pmatrix}$ **33** 77°

Exercise 4C PAGE 97

1 $-\mathbf{i} + 11\mathbf{j}$ **2** $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ **3** $\begin{pmatrix} -3 \\ 5 \end{pmatrix}$

4 Paths of particles do not cross in the *subsequent* motion. (If A was moving with the given velocity prior to $t = 0$ then, when $t = -3$ particle A was at $7\mathbf{i} - 6\mathbf{j}$ and particle B reaches that point when $t = 4$.)

5 In the subsequent motion the paths of the particles do meet with both particles reaching the point with position vector $25\mathbf{i} + 10\mathbf{j}$ when $t = 6$. A collision is involved.

6 In the subsequent motion the paths of the particles do cross with particle A reaching the point with position vector $15\mathbf{i} + 12\mathbf{j}$ when $t = 7$, particle B being there when $t = 4$. A collision is not involved.

Exercise 4D PAGE 100

1 $\mathbf{r} \cdot (3\mathbf{i} + 4\mathbf{j}) = 18$

2 $\mathbf{r} \cdot (5\mathbf{i} - \mathbf{j}) = -12$

3 Points A, B, E and F lie on the line, C and D do not.

4 $u = 2, v = 10, w = 11, x = 8, y = 0, z = -4$

5 a $\mathbf{r} \cdot (5\mathbf{i} + 2\mathbf{j}) = 7$ **b** $5x + 2y = 7$

6 a $\mathbf{r} \cdot (2\mathbf{i} + 5\mathbf{j}) = -1$ **b** $2x + 5y = -1$

8 $8x + 5y = 7$

Exercise 4E PAGE 105

1 a $y = 2x - 8$ **b** $y = \frac{1}{x}$

c $y^2 = 4x$ **d** $y = (x^2 + 1)^2, x \geq 0$

2 a $y = 10 - 2x$ **b** $y = \frac{1}{x+1}$

c $y = x^2 + 2x + 5$ **d** $(x-2)^2 + \left(\frac{y-1}{2}\right)^2 = 1$

3 Parametric equations: $\begin{cases} x = 2\cos\theta \\ y = 3\sin\theta \end{cases}$

Cartesian equation: $9x^2 + 4y^2 = 36$

4 Parametric equations: $\begin{cases} x = -3\sec\theta \\ y = 2\tan\theta \end{cases}$

Cartesian equation: $4x^2 - 9y^2 = 36$

5 B, D, E.

6 a $|\mathbf{r}| = 25$

b A lies outside, B lies on, C lies inside, D lies on.

7 $x^2 + y^2 = 65^2$. $a = 39, b = -60$.

8 $|\mathbf{r} + 7\mathbf{i} - 4\mathbf{j}| = 4\sqrt{5}$. A lies on.

9 a $|\mathbf{r} - \mathbf{i} + 5\mathbf{j}| = 9$ **b** $|\mathbf{r} + 3\mathbf{i} - 4\mathbf{j}| = 10$

c $|\mathbf{r} + 12\mathbf{i} - 3\mathbf{j}| = 2\sqrt{3}$ **d** $|\mathbf{r} + 13\mathbf{i} + 2\mathbf{j}| = 4$

10 a $x^2 + y^2 - 4x - 6y = 12$ **b** $x^2 + y^2 + 8x - 4y = -13$

c $x^2 + y^2 - 8x + 6y = 24$

11 a $5, 6\mathbf{i} + 3\mathbf{j}$ **b** $6, 2\mathbf{i} - 3\mathbf{j}$ **c** $3, 3\mathbf{i} - 4\mathbf{j}$

d $20, 0\mathbf{i} + 0\mathbf{j}$ **e** $1.25, 0\mathbf{i} + 0\mathbf{j}$ **f** $7, 2\mathbf{i} - 3\mathbf{j}$

g $5, 3\mathbf{i} + 9\mathbf{j}$ **h** $11, -10\mathbf{i} + \mathbf{j}$

12 13

13 $\mathbf{r} = (2 + 3\lambda)\mathbf{i} + (-5 + 7\lambda)\mathbf{j}$

14 10. The circles have just one point in common because the distance between the centres equals the sum of the radii.

15 $2\sqrt{26}$. The circles have no points in common because the distance between the centres exceeds the sum of the radii.

16 $\begin{pmatrix} -3 \\ 12 \end{pmatrix}, \begin{pmatrix} 4 \\ 9 \end{pmatrix}$ **17** $-2\mathbf{i} + 6\mathbf{j}$

Exercise 4F PAGE 109

1 $\sqrt{5}$ km at 10:36 a.m. **2** $2\sqrt{5}$ m, 2.25

3 Approximately 1.8 metres. The snake probably catches the mouse.

4 $5\sqrt{13}$ cm, 5 **5** $3\sqrt{13}$ km

6 $\sqrt{17}$ m **7** $3\sqrt{29}$ units

8 5 units **9** $4\sqrt{2}$ units

Miscellaneous exercise four PAGE 111

2 **a** $y = |x + 3|$ **b** $y = |x - 3|$

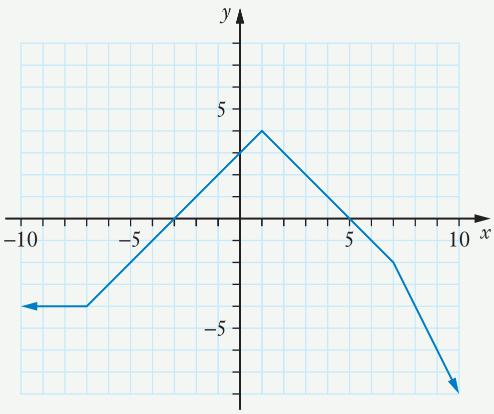
c $y = |3x - 6|$ **d** $y = |2x + 4|$

3 **a** $5, (7, -1)$ **b** $6, (7, 1)$

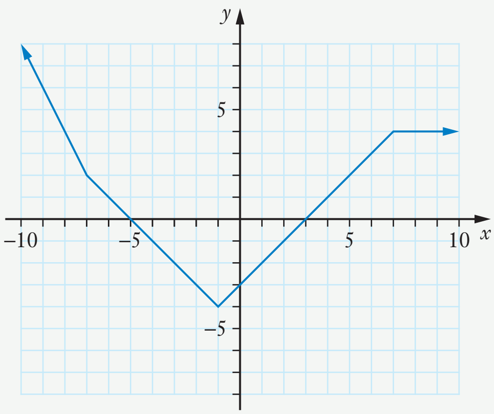
c $3\sqrt{2}, (0, 0)$ **d** $5\sqrt{3}, (1, -8)$

e $10, (-1, 7)$ **f** $15, (-5, 7)$

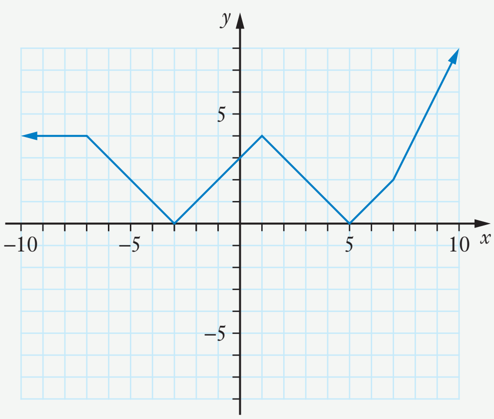
4 **a**



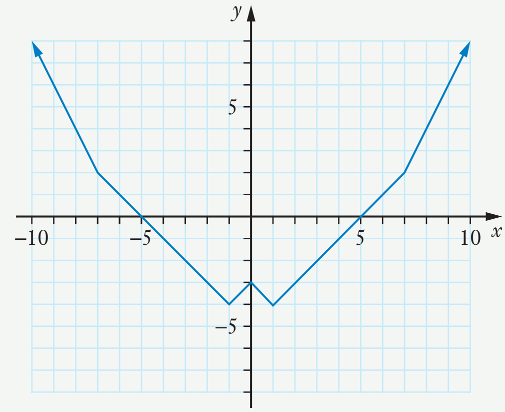
b



c



d



5 **a** $a = 3, b = 5, c = -2$ **b** $\frac{5}{3}, 1 + 2i, 1 - 2i$

6 **a** 0.8 **b** 0.5

c $\{x \in \mathbb{R}: x < 4\}$ **d** $\{y \in \mathbb{R}: y < 1\}$

e $f^{-1}(x) = 4 - \frac{1}{(1-x)^2}$, domain $\{x \in \mathbb{R}: x < 1\}$, range $\{y \in \mathbb{R}: y < 4\}$.

7 **a** $\sqrt{3} + i$ **b** $2\,\text{cis}\left(-\frac{2\pi}{3}\right)$

c $4\,\text{cis}\left(-\frac{\pi}{2}\right), -4i$ **d** $\text{cis}\left(\frac{5\pi}{6}\right), -\frac{\sqrt{3}}{2} + \frac{1}{2}i$

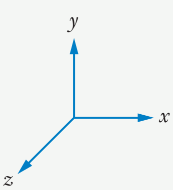
8 **a** Line cuts circle in two places, position vectors $20\mathbf{i} + 30\mathbf{j}$ and $40\mathbf{i} + 34\mathbf{j}$.

b Line neither touches not cuts the circle.

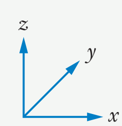
c Line is a tangent to the circle, point of contact $-2\mathbf{i} + 4\mathbf{j}$.

Exercise 5A PAGE 120

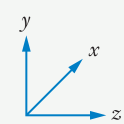
1 **a**



b



c



2 **a** $5\mathbf{i} + 14\mathbf{j} + 2\mathbf{k}$ **b** $-\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$

c $7\mathbf{i} + 20\mathbf{j} + 5\mathbf{k}$ **d** $10\mathbf{i} + 28\mathbf{j} + 4\mathbf{k}$

e 51 **f** 51

g 7 **h** 15

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3 **a** $\begin{pmatrix}1\\4\\7\end{pmatrix}$ **b** $\begin{pmatrix}-3\\4\\-1\end{pmatrix}$ **c** $\begin{pmatrix}0\\8\\10\end{pmatrix}$ **d** $\begin{pmatrix}2\\8\\14\end{pmatrix}$

e 10 **f** 10 **g** $\sqrt{26}$ **h** $\sqrt{66}$

4 **a** < 2, 2, – 3 > **b** < 3, 0, – 3 >

c < 1, 10, – 6 > **d** < 0, 6, – 3 >

e 7 **f** 42

g 17 **h** $\sqrt{17}$

5 **a** $\mathbf{i} - \mathbf{j} + 5\mathbf{k}$ **b** $-8\mathbf{i} - 3\mathbf{j}$

c $7\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}$ **d** $-7\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$

6 **a** $\begin{pmatrix}7\\1\\4\end{pmatrix}$ **b** $\begin{pmatrix}6\\-1\\4\end{pmatrix}$ **c** 57

7 **a** 7 **b** 15 **c** 8

d 85.6° (to 1 dp)

8 101° **9** 80° **10** 73°

11 **a** $\frac{1}{7}(2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k})$ **b** $\frac{5}{7}(2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k})$

c $\frac{7}{5}(3\mathbf{i} + 4\mathbf{k})$ **d** 31°

12 **a** Parallel **b** Neither

c Neither **d** Perpendicular

e Perpendicular **f** Neither

g Perpendicular

13 21 N

14 $3\mathbf{i} - 8\mathbf{k}$

15 $\mathbf{a} = \begin{pmatrix}5\\2\\-1\end{pmatrix}, \mathbf{b} = \begin{pmatrix}2\\-1\\3\end{pmatrix}$

16 $p = 2, q = -4, r = 6$

17 **a** $-2\mathbf{i} + 9\mathbf{k}$ **b** $4\mathbf{j} + 7\mathbf{k}$ **c** $\sqrt{93}$ m **d** 4.5

19 $\begin{pmatrix}1\\6\\2\end{pmatrix}$

20 $5\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}$

21 $8\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}$

23 To 1 dp: 57.7°, 36.7°, 74.5°

24 $\mathbf{d} = \mathbf{a} - \mathbf{b} + 2\mathbf{c}$, $\mathbf{e} = \mathbf{a} - 2\mathbf{b} + \mathbf{c}$, $\mathbf{f} = -2\mathbf{a} - \mathbf{b} + \mathbf{c}$

25 **a** $\overrightarrow{DC} = 10\mathbf{i}$, $\overrightarrow{DB} = 10\mathbf{i} + 4\mathbf{k}$, $\overrightarrow{DI} = 3\mathbf{j} + \mathbf{k}$

b 83°

26 To 1 dp: **a** 60.8° **b** 73.0°

27 **a** $\overrightarrow{AB} = \begin{pmatrix}6\\2\\1\end{pmatrix}$, $\overrightarrow{BC} = \begin{pmatrix}-1\\-2\\3\end{pmatrix}$, $\overrightarrow{AC} = \begin{pmatrix}5\\0\\4\end{pmatrix}$

c 41

d To nearest degree: ∠A = 34°, ∠B = ∠C = 73°

Exercise 5B PAGE 126

2 $\mathbf{a} \times \mathbf{b} = 6\mathbf{i} - \mathbf{j} + 9\mathbf{k}$ **3** $\mathbf{c} \times \mathbf{d} = -\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$

4 $\mathbf{p} \times \mathbf{q} = 6\mathbf{j} + 9\mathbf{k}$ **5** $\mathbf{i} \times \mathbf{j} = \mathbf{k}$

6 **a** $\mathbf{a} \times \mathbf{b} = 2\mathbf{j} + 4\mathbf{k}$, $|\mathbf{a} \times \mathbf{b}| = 2\sqrt{5}$

b $|\mathbf{a} \times \mathbf{b}| = 2\sqrt{5}$

7 $\frac{1}{\sqrt{3}}(\mathbf{i} + \mathbf{j} + \mathbf{k})$ [or $-\frac{1}{\sqrt{3}}(\mathbf{i} + \mathbf{j} + \mathbf{k})$]

8 $\frac{1}{\sqrt{17}}(2\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$ [or $-\frac{1}{\sqrt{17}}(2\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$]

Exercise 5C PAGE 134

1 **a** $\mathbf{r} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k} + \lambda(2\mathbf{i} - \mathbf{j} + 2\mathbf{k})$

b $x = 3 + 2\lambda, y = 2 - \lambda, z = -1 + 2\lambda$

2 **a** $\mathbf{r} = 4\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \lambda(\mathbf{i} + \mathbf{j} + 2\mathbf{k})$

b $x = 4 + \lambda, y = 2 + \lambda, z = 3 + 2\lambda$

3 $\mathbf{r}.(3\mathbf{i} - \mathbf{j} + 5\mathbf{k}) = 19$

4 $\mathbf{r}.\begin{pmatrix}5\\1\\3\end{pmatrix} = 2$

5 $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j}) + \mu(3\mathbf{i} - 4\mathbf{j} + 6\mathbf{k})$

6 $\mathbf{r} = \begin{pmatrix}-3\\2\\-1\end{pmatrix} + \lambda\begin{pmatrix}2\\0\\-3\end{pmatrix} + \mu\begin{pmatrix}1\\-3\\2\end{pmatrix}$

7 $a = -7, b = 4$

8 $3x + 2y - z = 21$

9 $\mathbf{r}.(2\mathbf{i} - 3\mathbf{j} + 7\mathbf{k}) = 5$

11 Point of intersection has position vector $\begin{pmatrix}-2\\2\\4\end{pmatrix}$.

12 Point of intersection has position vector $-4\mathbf{i} + 13\mathbf{j} + 13\mathbf{k}$.

13 **b** $3\mathbf{i} + 6\mathbf{j} - 7\mathbf{k}$, 90°

15 Collision occurs when $t = 7$, at point with position vector $\begin{pmatrix}25\\-50\\30\end{pmatrix}$.

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16 Can be written in many ways.

One possibility is $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -3 \\ -5 \end{pmatrix}$.

Cartesian equation: $x + 2y - z = 5$

Scalar product form: $\mathbf{r} \cdot \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} = 5$

17 $9\mathbf{i} - 8\mathbf{j} + 15\mathbf{k}$

18 3.25

19 $(720\mathbf{i} + 600\mathbf{j} - 6\mathbf{k})$ km/h

20 **b** The planes are 9 units apart.

21 Minimum separation distance is 7 metres and it occurs when $t = 10$.

Exercise 5D PAGE 138

1	Centre (0, 0, 0),	radius 16
2	Centre (0, 0, 0),	radius 10
3	Centre (1, 1, 1),	radius 25
4	Centre (2, –3, 4),	radius 18
5	Centre (3, –1, 2),	radius $\sqrt{10}$
6	Centre (–4, 1, 0),	radius 5
7	Centre (0, 4, 0),	radius $5\sqrt{2}$
8	Centre (1, –3, 0),	radius 5
9	Centre (0, 3, –1),	radius 11
10	Centre (–4, 1, –1),	radius 5

11 Outside **12** On

13 Outside **14** Inside

15 Outside **16** On

17 Inside **18** On

19 $a = 4, b = 6, c = 4$ **20** $\begin{pmatrix} 2 \\ 6 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 6 \\ -4 \\ 7 \end{pmatrix}$

21 $10\mathbf{i} - \mathbf{j}$ and $6\mathbf{i} - 2\mathbf{j} + 9\mathbf{k}$

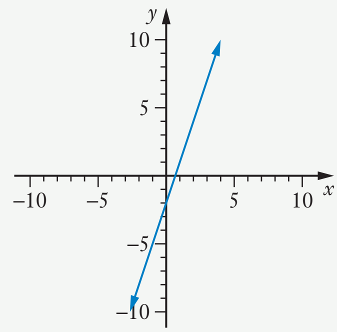
22 $7\mathbf{i} - \mathbf{j} + \mathbf{k}$ **23** $\begin{pmatrix} 4 \\ -2 \\ 5 \end{pmatrix}$

Exercise 5E PAGE 141

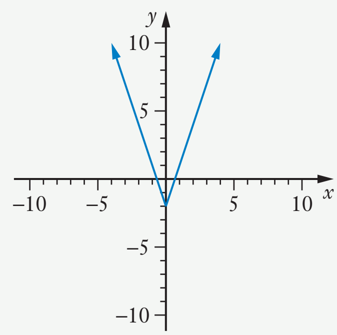
1 $\dfrac{2\sqrt{42}}{7}$ **2** $\dfrac{\sqrt{195}}{15}$ **3** $\dfrac{3\sqrt{26}}{26}$

Miscellaneous exercise five PAGE 142

1 **a** $f(3) = 7$ **b** $f(-3) = -11$ **c** $g(3) = 7$
d $g(-3) = 7$ **e** $f(5) = 13$ **f** $g(-5) = 13$
g Graph of $y = f(x)$, i.e. $y = 3x - 2$



Graph of $y = f(|x|)$, i.e. $y = 3|x| - 2$



2 **a** P has coordinates (5, –1).
b The circle has a radius of 5 units.
c The vector equation of the circle is $|\mathbf{r} - (5\mathbf{i} - \mathbf{j})| = 5$

3 **a** 7, (3, –2) **b** 11, (2, 7)
c 4, (3, –2) **d** $2\sqrt{5}$, (–1, –7)
e 5, (4, 2) **f** 10, (–3, 7)

4 75°

5 **a** $\{y \in \mathbb{R}: y \geq 0\}$ **b** $\{y \in \mathbb{R}: y \geq 3\}$
c $\{y \in \mathbb{R}: y \geq 0\}$ **d** $\{y \in \mathbb{R}: y \geq 0\}$
e $\{y \in \mathbb{R}: y \geq 3\}$ **f** $\{y \in \mathbb{R}: y \geq 0\}$

6 $a < 26$

7 $f \circ g(x) = \dfrac{3}{2x - 1}$, Domain $\{x \in \mathbb{R}: x \neq 0.5\}$,
Range $\{y \in \mathbb{R}: y \neq 0\}$

$g \circ f(x) = \dfrac{6}{x} - 1$, Domain $\{x \in \mathbb{R}: x \neq 0\}$,
Range $\{y \in \mathbb{R}: y \neq -1\}$

8 $f \circ g(x) = \sqrt{x^2 + 4}$, Domain $\mathbb{R}$, Range $\{y \in \mathbb{R}: y \geq 2\}$
$g \circ f(x) = x + 4$, Domain $\{x \in \mathbb{R}: x \geq -3\}$,
Range $\{y \in \mathbb{R}: y \geq 1\}$

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9 **a** $8\operatorname{cis}\left(-\frac{\pi}{6}\right)$ **b** $8\sqrt{3}\operatorname{cis}0$ **c** $8\operatorname{cis}\left(\frac{\pi}{2}\right)$

d $64\operatorname{cis}0$ **e** $1\operatorname{cis}\left(\frac{\pi}{3}\right)$

10 **a** 1 **b** 7 **c** $4\sqrt{2}-3$

d $4\sqrt{2}+3$ **e** $4\sqrt{2}+3$

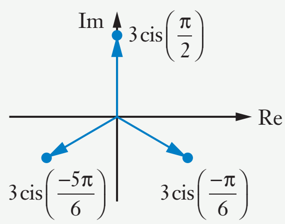
11 **a** $6\operatorname{cis}\left(\frac{5\pi}{6}\right)$ **b** $6\operatorname{cis}\left(-\frac{2\pi}{3}\right)$ **c** $6\operatorname{cis}\left(\frac{\pi}{6}\right)$

d $1.5\operatorname{cis}\left(\frac{\pi}{2}\right)$ **e** $3\operatorname{cis}\left(-\frac{2\pi}{3}\right)$ **f** $2\operatorname{cis}\left(\frac{\pi}{3}\right)$

g $3\operatorname{cis}\left(-\frac{5\pi}{6}\right)$ **h** $6\operatorname{cis}\left(-\frac{\pi}{6}\right)$ **i** $6\operatorname{cis}\left(-\frac{\pi}{6}\right)$

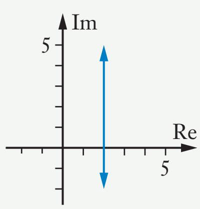
j $72\operatorname{cis}\left(-\frac{\pi}{3}\right)$

12

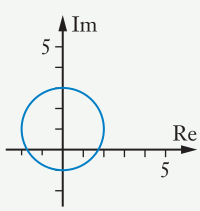


13 $\frac{5^4}{2^4}\operatorname{cis}\left(\frac{2\pi}{3}\right)$, $\frac{3^4}{2^4}\operatorname{cis}\left(\frac{2\pi}{3}\right)$

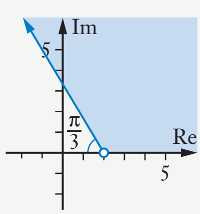
14 **a**



b



c



15 $9\mathbf{j}+\mathbf{k}$

16 **a** The lines are parallel. **b** The lines are skew.

c The lines intersect. **d** The lines are parallel.

17 13, 52 m

19 ~56 m, $(-192\mathbf{i}+216\mathbf{j}+16\mathbf{k})$ m/s

Exercise 6A PAGE 155

1 $x=3, y=-2, z=5$ **2** $x=4, y=7, z=-2$

3 $x=-1, y=5, z=1$ **4** $x=1, y=4, z=-3$

5 $x=3, y=4, z=6$ **6** $x=1, y=-2, z=-3$

7 $\begin{bmatrix} 3 & 2 & 10 \\ 1 & -4 & 8 \end{bmatrix}$ **8** $\begin{bmatrix} -1 & 5 & 12 \\ 2 & 3 & 2 \end{bmatrix}$

9 $\begin{bmatrix} 1 & 4 & 3 & 18 \\ 3 & 1 & 2 & 11 \\ 5 & 2 & 1 & 12 \end{bmatrix}$ **10** $\begin{bmatrix} 2 & 0 & 3 & 14 \\ 4 & 1 & -1 & 0 \\ 2 & 1 & 6 & 26 \end{bmatrix}$

11 $\begin{bmatrix} 3 & 2 & 0 & 8 \\ 1 & 0 & 2 & 8 \\ 0 & 2 & -1 & -1 \end{bmatrix}$ **12** $\begin{bmatrix} 1 & 3 & -5 & 2 \\ 2 & 1 & 7 & 37 \\ -1 & 0 & 1 & 3 \end{bmatrix}$

13 $x=7, y=9$ **14** $x=5, y=-2$

15 $x=3, y=1, z=2$ **16** $x=4, y=3, z=-1$

17 $x=7, y=0, z=-2$ **18** $x=3, y=-1, z=-2$

19 $x=5, y=1, z=-8$ **20** $x=3, y=1, z=3$

21 $x=2, y=-3, z=4$ **22** $x=3, y=1, z=-2$

23 $x=-5, y=11, z=0$ **24** $w=1, x=-3, y=2, z=1$

25 $5x+3y=270, x+2y=110, x=30, y=40$

26 $5p+10q+4r=160, 2p+q+4r=94, p+2q+2r=56$

4 P tablets, 6 Q tablets and 20 R tablets.

27 **a** $5x+3y+8z=6100, x+5y+z=1800,$
$4x+2y+z=2100$

b $x=300, y=200, z=500$

Exercise 6B PAGE 165

1 0 **2** 2 **3** −0.5

4 −3 **5** 1.5 **6** $k \neq 2$

7 2 **8** −1 **9** 3

10 −5 **11** 3 **12** 6

13 0 **14** −0.5 **15** −2

16 0 **17** 0

18 Infinite solutions for all values of k. Thus k can take any value.

19 −1 **20** 3

21 **a** $p=1.5, q=10$ **b** $p=1.5, q\neq 10$

c $p\neq 1.5$, no restriction on q.

22 **a** $p=6, q=9$ **b** $p=6, q\neq 9$

c $p\neq 6$, no restriction on q.

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23 p = 9, q = 1

24 p = −1, q = 5

25 p ≠ −1

26 p ≠ 5

27 Infinite solutions

28 **a** k ≠ 0.5, no restriction on m
b k = 0.5, m ≠ −1
c k = 0.5, m = −1

29 **a** no solution
b infinite solutions
c unique solution: $x = 3, y = -1, z = 4$

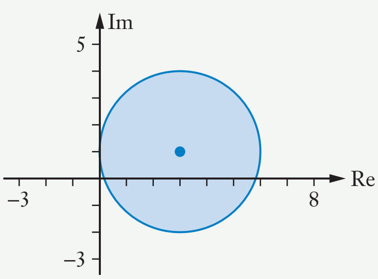
30 **a** k ≠ 2, no restriction on m
b $k = 2, m \neq -\frac{4}{3}$
c $k = 2, m = -\frac{4}{3}$

Miscellaneous exercise six PAGE 167

2 **a** $-8 < a < 2$
b −8 or 2

3 **a** $P_1\,(0, a)$, $P_2\,(0, b)$
b $a > b$
c $P_4\,(a, 0)$, $P_6\,(2b, 0)$
d $P_3\,(2a - 2b, 2b - a)$, $P_5\left(\frac{2a+2b}{3}, \frac{2b-a}{3}\right)$
e A, E, G
f C
g B, F
h D

4

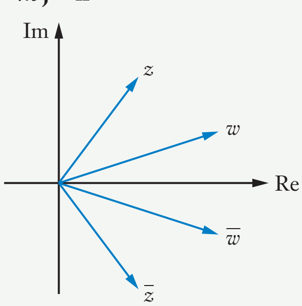


5 **a** $10 \operatorname{cis} \frac{-5\pi}{6}$
b $-3\sqrt{2} + 3\sqrt{2}i$

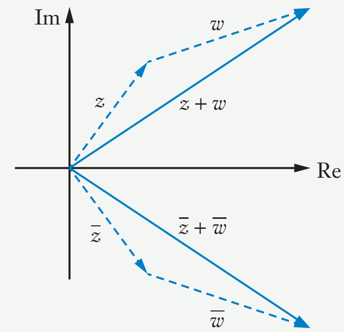
7 **a** $z = \sqrt{2} \operatorname{cis}\left(-\frac{\pi}{4}\right)$
b $128i$

8 $5\mathbf{i} + 4.5\mathbf{j} - \mathbf{k}$

9 **a**



b



$\overline{z + w}$ is the reflection of $(z + w)$ in the real axis.
But, from the diagram, $\bar{z} + \bar{w}$ is a reflection of $(z + w)$ in the real axis.

Hence $\overline{z + w} = \bar{z} + \bar{w}$.

c Justification not shown here. Compare your answer to those of others in your class.

10 $16 \operatorname{cis} 160°$, $2 \operatorname{cis} 130°$, $2 \operatorname{cis}(-50°)$, $2 \operatorname{cis}(-140°)$

11 **a** Domain $\{x \in \mathbb{R}: x > 3\}$, Range $(y \in \mathbb{R}: y > 4\}$.
b $f^{-1}(x) = \frac{1}{(x-4)^2} + 3$, Domain $\{x \in \mathbb{R}: x > 4\}$, Range $(y \in \mathbb{R}: y > 3\}$.

12 $\cos 4\theta = \cos^4\theta - 6\cos^2\theta \sin^2\theta + \sin^4\theta$,
$\sin 4\theta = 4\cos^3\theta \sin\theta - 4\cos\theta \sin^3\theta$

13 The planes are 8 units apart.

14 The shortest distance from the line to the origin is $\frac{\sqrt{26}}{2}$ units.

15 10

Exercise 7A PAGE 175

1 **a** j m
b $(54\mathbf{i} + 3\mathbf{j})$ m/s
c $15\sqrt{13}$ m/s
d $36\mathbf{i}$ m/s^2

2 **a** 10 m/s
b 12 m

3 **a** $\sqrt{5}$
b $\frac{5t - 1}{\sqrt{5t^2 - 2t + 1}}$

4 **a** $-0.25\mathbf{i} + 2\mathbf{j}$
b $0.25\mathbf{i}$
c $2.5\mathbf{i} + 5\mathbf{j}$

5 **a** 7
b 2.5

6 **a** $2\mathbf{i} + e\mathbf{j}$
b $0.1e\mathbf{j}$
c $20\mathbf{i} + 10e\mathbf{j}$

7 **a** 15 m
b $(8\mathbf{i} + 6\mathbf{j})$ m/s
c 10 m/s
d 37°

8 **a** $4\sqrt{13}$ m/s
b $(18\mathbf{i} + 4\mathbf{j})$ m/s^2
c 176
d 15.3°

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9 **a** $\sqrt{10}$ m/s **b** $\sqrt{146}$ m/s
c $(2\mathbf{i} + 12\mathbf{j})$ m/s^2 **d** $(17\mathbf{i} + 124\mathbf{j} - 9\mathbf{k})$ m

10 **a** 8 **b** 3 **c** 1.5

11 $(3\mathbf{i} + 4\mathbf{j} + 6\mathbf{k})$ m, $2\mathbf{j}$ m/s, $2\mathbf{k}$ m/s^2

12 **a** $[2\mathbf{i} + (2t - 8)\mathbf{j}]$ m/s
b $[(2t+1)\mathbf{i} + (t^2 - 8t + 20)\mathbf{j}]$ m
c $\sqrt{74}$ m
d $2\sqrt{5}$ m/s
e 4, 4 m
f $4y = x^2 - 18x + 97$

13 **a** 2
b Does not cross the y-axis.

14 $[-2\pi\mathbf{i} + \pi^2\mathbf{j} + (e^\pi - \pi - 1)\mathbf{k}]$ m

15 **a** $\dfrac{\pi}{6}$
b $(6\cos 3t\mathbf{i} - 6\sin 3t\mathbf{j})$ m/s, $(-18\sin 3t\mathbf{i} - 18\cos 3t\mathbf{j})$ m/s^2

16 8 m

Exercise 7B PAGE 179

1 velocity $= (u + at)\mathbf{i}$ m/s,
position vector $= \left(ut + \dfrac{1}{2}at^2\right)\mathbf{i}$ m

2 $[14t\mathbf{i} + (35t - 4.9t^2)\mathbf{j}]$ m, 87.5 m, $y = \dfrac{5}{2}x - \dfrac{1}{40}x^2$

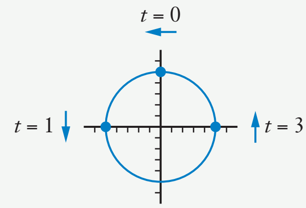
3 **a** $-10\mathbf{j}$ m/s^2
b $(40\mathbf{i} + 40\sqrt{3}\mathbf{j})$ m/s
c $[40t\mathbf{i} + (40\sqrt{3}t - 5t^2)\mathbf{j}]$ m
d $8\sqrt{3}$ s
e $320\sqrt{3}$ m

4 **a** $[42t\cos\theta\mathbf{i} + (42t\sin\theta - 4.9t^2)\mathbf{j}]$ m
b 20.9°, 69.1°

5 **a** $[u\cos\theta°\,\mathbf{i} + (u\sin\theta° - gt)\mathbf{j}]$ m/s
b $\left[ut\cos\theta°\mathbf{i} + \left(ut\sin\theta° - \dfrac{1}{2}gt^2\right)\mathbf{j}\right]$ m
c $\dfrac{2u\sin\theta°}{g}$ seconds
d $\dfrac{u^2\sin 2\theta°}{g}$ metres
e 45

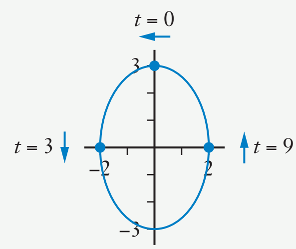
6 **a** $\mathbf{v}(t) = -\sin(0.5t)\mathbf{i} + \cos(0.5t)\mathbf{j}$,
$\mathbf{a}(t) = -0.5\cos(0.5t)\mathbf{i} - 0.5\sin(0.5t)\mathbf{j}$
b 1
c 0, velocity always perpendicular to acceleration.
d 0.25
e With $k > 0$, the acceleration is always directed towards (0, 0), the centre of the circle.

7 **a** $-5\sin\left(\dfrac{\pi}{2}t\right)\mathbf{i} + 5\cos\left(\dfrac{\pi}{2}t\right)\mathbf{j}$
b $5\mathbf{i}$
c



d $(5\mathbf{i} - 5\mathbf{j})$ m. This is the vector from $\mathbf{r}(0)$ to $\mathbf{r}(3)$. It is the displacement vector for $t = 0$ to $t = 3$.
$5\sqrt{2}$ m. This is the magnitude of the displacement from $t = 0$ to $t = 3$.
$\dfrac{15\pi}{2}$. This is the distance travelled from $t = 0$ to $t = 3$, i.e. three quarters of the circumference.

8 **a**



b $9x^2 + 4y^2 = 36$
c 1.20 rads
d Acceleration is always towards (0, 0). $k = \dfrac{\pi^2}{36}$

9 $\dfrac{49}{20}t(10\sqrt{3} - t)\mathbf{i} + \dfrac{49}{20}t(10 - \sqrt{3}t)\mathbf{j}$

10 **a** $(t - \sin t)\mathbf{i} + (1 - \cos t)\mathbf{j}$
b 2 m
c **i** $\mathbf{r} = 0\mathbf{i} + 0\mathbf{j}$, $\mathbf{v} = 0\mathbf{i} + 0\mathbf{j}$
ii $\mathbf{r} = (0.5\pi - 1)\mathbf{i} + \mathbf{j}$, $\mathbf{v} = \mathbf{i} + \mathbf{j}$
iii $\mathbf{r} = \pi\mathbf{i} + 2\mathbf{j}$, $\mathbf{v} = 2\mathbf{i} + 0\mathbf{j}$
iv $\mathbf{r} = (1.5\pi + 1)\mathbf{i} + \mathbf{j}$, $\mathbf{v} = \mathbf{i} - \mathbf{j}$

Miscellaneous exercise seven PAGE 182

1 $2\operatorname{cis}\dfrac{5\pi}{6}$ **2** $-3\sqrt{2} + 3\sqrt{2}i$

3 $f^{-1}(x) = \begin{cases} 4x & \text{for } x \le 0 \\ \sqrt{x} & \text{for } 0 < x < 9 \\ x \div 3 & \text{for } x \ge 9 \end{cases}$

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4 All of them.

5 $f^{-1}(x) = (x-3)^2 - 1$, Domain $\{x \in \mathbb{R}: x \geq 3\}$, Range $\{y \in \mathbb{R}: y \geq -1\}$.

6 **a** $\mathbf{p} = 2\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$

b $\mathbf{q} = -2\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$

c 85° (to nearest degree)

d 61° (to nearest degree)

e 42° (to nearest degree)

7 $\frac{2}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{1}{3}\mathbf{k}$ **8** $\begin{pmatrix} 2 \\ -10 \\ -13 \end{pmatrix}$

9 Domain $\{x \in \mathbb{R}: 0 \leq x \leq 0.64\}$, Range $\{y \in \mathbb{R}: 0 \leq y \leq 2\}$.

10 $(6\mathbf{i} + 8\mathbf{j})$ m/s, 10 m/s, $2\mathbf{j}$ m/s^2

11 (Graph not shown here – check with a graphic calculator display.) $-7 \leq x \leq 7$

12 **a** $2a$ **b** $2ib$ **c** $a^2 + b^2$

d $\frac{a^2 - b^2}{a^2 + b^2} + i\frac{2ab}{a^2 + b^2}$

13 $\begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix}$

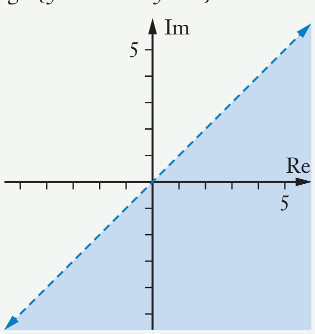
14 $f \circ g(x) = x - 9$, Domain $\{x \in \mathbb{R}: x \geq 9\}$, Range $\{y \in \mathbb{R}: y \geq 0\}$.

$g \circ f(x) = \sqrt{x^2 - 9}$, Domain $\{x \in \mathbb{R}: |x| \geq 3\}$, Range $\{y \in \mathbb{R}: y \geq 0\}$.

15 $f \circ g(x) = 9 - x$, Domain $\{x \in \mathbb{R}: x \leq 9\}$, Range $\{y \in \mathbb{R}: y \geq 0\}$.

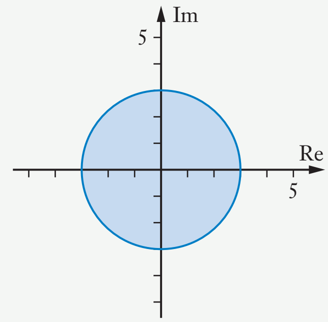
$g \circ f(x) = \sqrt{9 - x^2}$, Domain $\{x \in \mathbb{R}: -3 \leq x \leq 3\}$, Range $\{y \in \mathbb{R}: 0 \leq y \leq 3\}$.

16 **a**

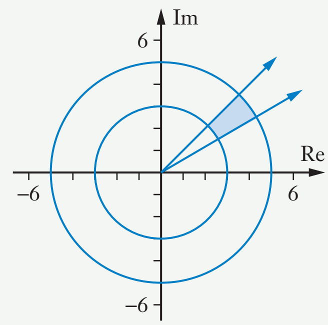


(Note the use of the dashed line to imply the line itself is not included.)

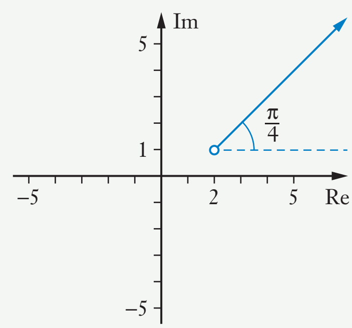
b



c



d



17 Points $z = x + iy$ satisfy the equation

$$\left(x + \frac{1}{3}\right)^2 + \left(y - \frac{4}{3}\right)^2 = \frac{8}{9},$$

i.e. a circle, centre $\left(-\frac{1}{3}, \frac{4}{3}\right)$ radius $\frac{2\sqrt{2}}{3}$.

18 Many possible answers, for example $\frac{1}{\sqrt{17}}\begin{pmatrix} 0 \\ -1 \\ 4 \end{pmatrix}$, $\frac{1}{9}\begin{pmatrix} 1 \\ 4 \\ -8 \end{pmatrix}$, but all must be of the form

$\frac{1}{\sqrt{a^2 + b^2 + c^2}}\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ with $-8a + 4b + c = 0$.

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19 **a** 1 **b** 16 **c** 17
d 9 **e** 0.082 **f** 0.708

20 L_1 and L_2 do not intersect.

21 $a = 3, b = 5, c = 3$, A (3, 1), B $\left(5, \frac{5}{3}\right)$.

22 $\mathbf{r} = \begin{pmatrix} 5 \\ -1 \\ 1 \end{pmatrix}$

23 $a = 9, b = -4, c = 7, d = 12, e = 9, f = -10$

24 $-\mathbf{a} + 2\mathbf{b} - 3\mathbf{c}$

25 $\mathbf{r}_F = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k}, c = 0, d = 9, e = 4$

26 **a** 1.07 radians
b $3\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$
c $\mathbf{r}.(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}) = 13$

28 $8\cos^4\theta - 8\cos^2\theta + 1$

29 $-b + ai, -a - bi, b - ai$

30 Full method should be shown, leading to
a $x = 3, y = 0, z = 4$ **b** $x = 3, y = 1, z = -1$

31 $4x^2 - y^2 = 16$ (For $x \geq 2$.)

32 $(-40\mathbf{i} - 16\mathbf{j} - 12\mathbf{k})$ m/s, 670 m

33 **a** **i** $q = 0$ **ii** $q \neq 0$
b $x = 2q + 0.5, y = 0.5, z = -q$
c $x = 1, y = \frac{3}{8}, z = -\frac{1}{8}$

34 **a** $2\mathbf{i} + (3\pi - 1)\mathbf{j}$
b 3 m/s, $[4\mathbf{i} + (0.75\pi - 1)\mathbf{j}]$ m

36 The shortest distance from the line to the given point is 3 units.

37 **a** $\left[ut\cos\theta\mathbf{i} + \left(ut\sin\theta - \frac{gt^2}{2}\right)\mathbf{j}\right]$ m
b 34.9° or 76.9°

38 Student's conclusion is incorrect. The last equation, $0x + 0y + 0z = 0$, is true for all values of x, y and z, perhaps suggesting infinite solutions. However, looking to the other lines we still have three other equations involving three unknowns so a unique solution may still be possible. Indeed from these we obtain $x = 1, y = -1$ and $z = 3$ (and of course these values also fit $0x + 0y + 0z = 0$). The conclusion the student should have made is that the system has a unique solution of $x = 1, y = -1$ and $z = 3$.

39 **a** $[30\mathbf{i} + (24 - 10t)\mathbf{j}]$ m/s **b** $[30t\mathbf{i} + (24t - 5t^2)\mathbf{j}]$ m
c 2.4 s **d** 4.5 s
e 28.8 m **f** 6.75

41 $-6\sqrt{6}$

42 If looking for x, y and z values that satisfy all of the equations, the first equation, $x + 3y - z = 3$, and second equation, $-x - 3y + z = 3$ (i.e. $x + 3y - z = -3$) are contradictory. Hence no solution. The two equations represent distinct parallel planes, hence no points in common.

43 **a** $p = -4, q = -1$
b $p = -4, q \neq -1$
c $m = -1, n = 2, p = -2, q = 5$

44 No. Closest distance to light is $\sqrt{42}$ m which is greater than 6 m.

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ANSWERS UNIT FOUR

Exercise 8A PAGE 199

1 $-\dfrac{y+8}{x+2}$

2 $\dfrac{6x-y+4}{x+1}$

3 $\dfrac{2(1+3xy)}{3(y^2-x^2)}$

4 $\dfrac{6x^2y+5}{2(y-x^3)}$

5 $\dfrac{2x+2y-3}{2(5y-x)}$

6 $\dfrac{2x+2y-1}{2(3y-x)}$

7 $\dfrac{9-2x}{2y}$

8 $\dfrac{2x}{9-2y}$

9 $\dfrac{9y-2x}{2y-9x}$

10 $\dfrac{9y+1-2x}{2y-9x-1}$

11 $\dfrac{\cos x}{\sin y}$

12 $\dfrac{2(x\cos y-5y)}{10x+x^2\sin y}$

13 8

14 0.125

15 −1.2

16 −3.2

17 $y=x$

18 $\dfrac{489}{212}$

19 $2xy+x^4y$

20 (1, 1.5), (1, −3)

21 (1, −3), (3, −3)

22 $\dfrac{dy}{dx}=\dfrac{2x+1}{1-3y^2}$. At (1, 0), $\dfrac{dy}{dx}=3$.

$\dfrac{d^2y}{dx^2}=\dfrac{2(1-3y^2)^2+6y(2x+1)^2}{(1-3y^2)^3}$. At (1, 0), $\dfrac{d^2y}{dx^2}=2$.

23 $6y=4\sqrt{3}x+\pi-4\sqrt{3}$

24 $\dfrac{dy}{dx}=\dfrac{\sin x}{2y-3}$, $\dfrac{d^2y}{dx^2}=\dfrac{(2y-3)^2\cos x-2\sin^2 x}{(2y-3)^3}$

25 $\dfrac{dy}{dx}=\dfrac{x+1}{\cos y}$. At $\left(-2,\dfrac{\pi}{6}\right)$, $\dfrac{dy}{dx}=-\dfrac{2\sqrt{3}}{3}$.

$\dfrac{d^2y}{dx^2}=\dfrac{\cos^2 y+(x+1)^2\sin y}{\cos^3 y}$.

At $\left(-2,\dfrac{\pi}{6}\right)$, $\dfrac{d^2y}{dx^2}=\dfrac{10\sqrt{3}}{9}$.

26 $\left(\dfrac{\sqrt{3}}{2},\dfrac{3\sqrt{3}}{2}\right)$ and $\left(-\dfrac{\sqrt{3}}{2},-\dfrac{3\sqrt{3}}{2}\right)$

Exercise 8B PAGE 201

1 **a** $6\cos 2t$ **b** $-10\sin 5t$ **c** $-\dfrac{5\sin 5t}{3\cos 2t}$

2 **a** $2\sin t\cos t$ **b** $-3\sin 3t$ **c** $-\dfrac{3\sin 3t}{\sin 2t}$

3 $\dfrac{2t}{3}$

4 $\dfrac{3}{2t}$

5 $\dfrac{2(t+1)}{15t^2}$

6 $-\dfrac{1}{6(t+1)^3}$

7 $\dfrac{t-1}{t}$

8 $\dfrac{2(t-1)^2}{(t+1)^2}$

9 −1.5

10 −36

11 (14, −16), (2, 16)

12 **a** $\dfrac{\cos 2t}{\cos t}$ **b** $(2,\sqrt{3})$, $\dfrac{1}{\sqrt{3}}$

c $\dfrac{\pi}{4}$, $\dfrac{3\pi}{4}$, $\dfrac{5\pi}{4}$, $\dfrac{7\pi}{4}$

13 **a** $\dfrac{t^2-2}{2t^2+1}$ **b** $\dfrac{10t^3}{(2t^2+1)^3}$

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Exercise 8C PAGE 206

1 200 **2** 1.5 **3** 2

4 **a** 0.5 **b** 36

5 0.15 **6** 0.25 **7** 0.36

8 –8 **9** 0.25 cm^2/s **10** $\frac{1}{\sqrt{2}}$ cm^2/s

11 Decreasing at 0.075 cm/s

12 0.16 cm^2/s **13** 6 cm^2/s

14 $60\sqrt{3}$ cm^2/min

15 $0.4\,\pi r^2$ cm^3/s **a** 10π cm^3/s **b** 10 cm

16 **a** 12 cm^2/s **b** 30 cm^3/s

17 **a** 80 cm/min **b** 40 cm/min

c 16 cm/min

18 **a** $3r^2$ cm^3/s **b** $4.8r$ cm^2/s

19 **a** 1.6 cm/s **b** 0.8 cm/s

20 **a** $4x(2x^2-3)$ m/s^2 **b** 5 m/s, 40 m/s^2

21 $2\sqrt{3}$ cm^2/s

22 **a** **i** 4 cm/s **ii** 1 cm/s

b 22 mm/s

23 **a** $\frac{1}{32\pi}$ m/min **b** $\frac{1}{4\pi}$ m/min

24 1090

25 **a** π cm^2/s **b** 2.5π cm^3/s

26 **a** 4.8π cm^2/s **b** 14π cm^3/s

27 $\frac{25}{6}$ cm/s **28** 27 mm/min

29 **a** shortening at 0.6 m/s **b** 2 m/s

30 **a** lengthening at 1 m/s **b** 3 m/s

31 $-\frac{1}{2\sqrt{3}}$ cm/s **32** $\frac{180}{13}$ m/s ($\approx$ 13.8 m/s)

33 4 m/s **34** $\frac{89\pi}{2}$ m/s ($\approx$ 139.8 m/s)

Exercise 8D PAGE 212

1 $0.7, f(5.01)-f(5)=0.701\,501$

2 $0.015,\ f\left(\frac{\pi}{9}+0.01\right)-f\left(\frac{\pi}{9}\right)=0.0146$ (to 4 decimal places)

3 $0.011\,25,\ f\left(\frac{\pi}{3}+0.001\right)-f\left(\frac{\pi}{3}\right)=0.011\,265\,9$ (to 7 decimal places)

4 $\frac{10}{\sqrt{x}}$

a \$2 per unit **b** \$1 per unit

c \$0.50 per unit

5 $750-30x+\frac{3x^2}{10}$

a \$120 per tonne **b** \$30 per tonne

c \$750 per tonne

6 \$10 per unit. It will cost approximately \$10 to produce the 11th item.

7 **a** 12 cm^2 **b** 15 cm^3

Exercise 8E PAGE 213

3 **a** $x^x(1+\ln x)$ **b** $2x^{2x}(1+\ln x)$

c $\frac{x^{\cos x}(\cos x - x\sin x\ln x)}{x}$

d $-\frac{3}{\sqrt{(3x+1)(3x-1)^3}}$

Miscellaneous exercise eight PAGE 214

1 **a** $\frac{8}{(3-2x)^2}$ **b** $6\sin^2(2x+1)\cos(2x+1)$

c $\frac{5-6xy}{3(x^2+y^2)}$ **d** $\frac{4t^3}{2t+3}$

2 $4y=-3x+25$

3 **a** $2y^3$ **b** $\frac{2y^3(5-y^3)}{(2y^3+5)^3}$

4 **a** $\frac{40}{3}$ m/sec $\uparrow$, $\frac{4\sqrt{3}}{9}$ m/s^2 $\uparrow$.

b 40 m/sec $\uparrow$, $4\sqrt{3}$ m/s^2 $\uparrow$.

Exercise 9A PAGE 219

1 $5(x^2-3)^6+c$ **2** $-(1-2x)^4(1+8x)+c$

3 $\frac{2}{63}(3x+1)^6(18x-1)+c$ **4** $\frac{1}{4}(2x^2-1)^6+c$

5 $\frac{1}{3}(3x^2+1)^6+c$ **6** $\frac{1}{7}(x-2)^6(3x+1)+c$

7 $-(4x+3)(3-x)^4+c$ **8** $-\frac{1}{42}(5-2x)^6(12x+5)+c$

9 $\frac{1}{4}(2x+3)^4(8x-3)+c$ **10** $\frac{4}{15}(3x+1)^{\frac{3}{2}}(9x-2)+c$

11 $2\sqrt{3x^2+5}+c$ **12** $-(x+1)\sqrt{1-2x}+c$

13 $\frac{2}{3}\sin^6 2x+c$ **14** $-\frac{9}{8}\cos^8 3x+c$

15 $-3\cos(x^2+4)+c$ **16** $\frac{1}{84}(2x+1)^6(24x+19)+c$

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Exercise 9B PAGE 220

1 $\frac{1}{2}x^2 - \frac{1}{3}\cos 3x + c$

2 $2x + c$

3 $-\frac{1}{8}\cos 8x + c$

4 $\frac{1}{2}\sin 2x + c$ $\left(\text{or } \frac{1}{2}(\cos x + \sin x)^2 + c\right)$

5 $\frac{2}{5}x^{\frac{5}{2}} + \frac{2}{3}x^{\frac{3}{2}} + c$

6 $-2\cos(x^2) + c$

7 $-4\cos(x^2 - 3) + c$

8 $\frac{16}{3}(1+3x)^{\frac{3}{2}} + c$

9 $\frac{2}{9}(1+3x)^{\frac{3}{2}}(9x-2) + c$

10 $\frac{1}{10}\sin^5 2x + c$

11 $\frac{1}{28}(2x+7)^6(12x-7) + c$

12 $\frac{1}{2}(2x+7)^6 + c$

13 $x^3 - 2x + c$

14 $\frac{1}{12}(3x^2-2)^8 + c$

15 $\sin x - \frac{1}{2}\cos 2x + c$

16 $\frac{1}{54}(3x-2)^8(12x+1) + c$

17 $\frac{1}{2}x^2 + c$

18 $6\sqrt{1+2x} + c$

19 $2(x-1)\sqrt{1+2x} + c$

20 $\frac{1}{9}(x^2+x+1)^9 + c$

21 $-12\cos(x^2+3) + c$

22 $\frac{3}{28}(x-5)^{\frac{4}{3}}(8x+37) + c$

23 $\frac{1}{3}(\sqrt{x}+5)^6 + c$

24 $\frac{(2x-1)^6}{3} + c$

25 $\frac{1}{42}(2x-1)^6(12x+1) + c$

26 $-\frac{\cos^4 6x}{24} + c$

27 $6\sqrt{x^2-3} + c$

28 $-\frac{\cos 4x}{8} + c$ or $\frac{\sin^2 2x}{4} + c$

29 $\frac{1}{168}(2x-1)^6(84x^2+12x+1) + c$

Exercise 9C PAGE 222

1 160

2 113.6

3 125

4 2

5 9.28

6 $12\frac{2}{3}$

7 8 square units

8 72.9 square units

Exercise 9D PAGE 225

1 $\frac{1}{18}\sin 9x + \frac{1}{2}\sin x + c$

2 $\frac{1}{12}\sin 6x - \frac{1}{16}\sin 8x + c$

3 $\frac{1}{5}\sin^5 x + c$

4 $\frac{3}{2}\sin^4 x + c$

5 $-\cos x + \frac{1}{3}\cos^3 x + c$

6 $\sin x - \frac{1}{3}\sin^3 x + c$

7 $\sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x + c$

8 $\frac{x}{2} + \frac{\sin 2x}{4} + c$

9 $\frac{x}{2} - \frac{\sin 2x}{4} + c$

10 $3x - 2\sin 2x + \frac{1}{4}\sin 4x + c$

11 $x + c$

12 $\frac{1}{2}\sin 2x + c$

13 $-\cos x + \frac{1}{3}\cos^3 x + \frac{\sin 2x}{4} + \frac{x}{2} + c$

14 $-\frac{\cos 2x}{2} + c$ (or $\sin^2 x + c$ or $-\cos^2 x + c$)

15 $-\frac{1}{3}\cos^3 x + \frac{1}{5}\cos^5 x + c$

16 $\frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + c$

17 $\frac{1}{3}\tan 3x - x + c$

18 $\tan x + c$

19 $\tan x - x + c$

20 $\frac{1}{5}\tan^5 x + c$

21 $2\pi^2$ square units

22 a $\mathbf{r} = (3 + 2t - \sin 2t)\mathbf{i} + (1 - t + \tan t)\mathbf{j}$

b $\mathbf{r}\left(\frac{\pi}{4}\right) = \left(2 + \frac{\pi}{2}\right)\mathbf{i} + \left(2 - \frac{\pi}{4}\right)\mathbf{j}$

Exercise 9E PAGE 231

1 $7\ln|x| + c$

2 $x^3 - 4\ln|x| + c$

3 $4\ln(x^2+6) + c$

4 $-\frac{1}{2}\ln|\cos 2x| + c$

5 $x + 2\ln|x| + c$

6 $x - 2\ln|x+2| + c$

7 $2x - 3\ln|x| + c$

8 $\frac{x}{2} + \frac{3}{4}\ln|2x-3| + c$

9 $\frac{x^2}{2} + x - 2\ln|x+3| + c$

10 $3\ln|x| + 2\ln|x+1| + c$

11 $3\ln|x+2| + \ln|x-3| + c$

12 $3\ln|x-1| + \ln(x^2+6) + c$

13 $5\ln|x+1| + \ln|x^2+x-1| + c$

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14 $3\ln|x+1| + 2\ln|x-1| + \dfrac{4}{x-1} + c$

15 $2\ln|2x+1| + 2\ln|x-3| + \dfrac{5}{x-3} + c$

Exercise 9F PAGE 233

1 $\dfrac{32\pi}{5}$ units^3 **2** $\dfrac{9\pi}{5}$ units^3 **3** $\dfrac{15\pi}{2}$ units^3

4 $\dfrac{109\pi}{3}$ units^3

5 a $\dfrac{\pi}{2}$ units^3 **b** $\dfrac{\pi}{6}$ units^3

6 $\dfrac{78\pi}{5}$ units^3 **7** 18π units^3 **8** 2π units^3

9 $\dfrac{\pi^2}{2}$ units^3 **10** $\dfrac{2\pi}{15}$ units^3 **11** $\dfrac{24\pi}{5}$ units^3

12 $4\pi^2$ units^3 **13** $\dfrac{4}{3}\pi r^3$ **14** $\dfrac{1}{3}\pi r^2 h$

15 2π units^3 **16** $\dfrac{7\pi}{15}$ units^3 **17** 108π cm^3

18 $\dfrac{7\pi}{2}$ units^3 **19** $\dfrac{\pi^2}{16}$ m^3, $\dfrac{\pi^2}{16}$ m^3

20 160π units^3

21 $2\pi\int_a^b xy\,dx$ **a** $\dfrac{15\pi}{2}$ units^3 **b** $\dfrac{199\pi}{5}$ units^3

22 $2\pi\int_a^b xy\,dy$ **a** 2π units^3 **b** $\dfrac{7\pi}{3}$ units^3

Miscellaneous exercise nine PAGE 239

1 $6(2x+1)^2$ **2** $-12\sin 3x + 12\cos 4x$

3 $\dfrac{\sin^3 x(4x\cos x - \sin x)}{x^2}$ **4** $\dfrac{2+\sin x + 2\cos x}{(1+\cos x)^2}$

5 $\dfrac{2\cos 2x}{(1+\sin 2x)^2}$ **6** $\dfrac{6x-5y}{5x+6y^2}$

7 $\dfrac{-12t^2}{6t-5}$ **8** $\dfrac{\cos y - y\cos x}{\sin x + x\sin y}$

9 $a=1, b=6, \ln|x-1| + 6\ln|x+1| + c$

10 a $\dfrac{1}{2}\sin 8x + c$

b $\dfrac{1}{6}(3+x^2)^6 + c$

c $\dfrac{3}{28}(x+3)^{\frac{4}{3}}(41-12x) + c$

d $\dfrac{1}{12}\sin^6 2x + c$

e $\dfrac{1}{2}x - \dfrac{1}{2}\sin x + c$

f $2\sin\dfrac{x}{2} - \dfrac{2}{3}\sin^3\dfrac{x}{2} + c$

g $\dfrac{1}{6}\cos^3 2x - \dfrac{1}{2}\cos 2x + c$

h $-4\cos^3 x + c$

i $-4\cos^3 x + 6\cos x + c$

11 a $y = 1.75x - 0.5$ **b** $9y + 4x = 35$

12 $(\pi \ln 4)$ units^3 **13** 24 cm^2/s

15 $-0.5 + \ln 3$ **16** $\dfrac{4}{5\pi}$ cm/sec

17 a $\dfrac{13}{3}$ m/s **b** $\dfrac{13}{3}$ m/s **c** $\dfrac{5}{3}$ m/s

Exercise 10A PAGE 246

1 $y = 4x^2 - 5x + c$ **2** $y = 4x^{\frac{3}{2}} + c$

3 $4y^2 = 2x^2 - x + c$ **4** $\dfrac{3y^2}{2} = -\dfrac{5}{x} + c$

5 $7y^2 = -\dfrac{1}{x} + c$ **6** $2\cos 2y = \dfrac{5}{x} + c$

7 $y^2 - 3y = 4x^2 + x + c$ **8** $2y^2 - 5y = x^2 - x^3 + c$

9 $\sin y = -\dfrac{1}{x} + c$ **10** $(y^2+1)^6 = 3x^2 + c$

11 $y = 3x^2 + 1$ **12** $y^2 = \dfrac{13}{3} - \dfrac{5}{3x}$

13 $2y + \sin y = x^2 + 3x + \pi - 3$

14 $y^2 + 3y = x^4 + 4x^2 + 5$

15 When $s = 3$, $v = 4\sqrt{7}$

16 a $a = 1$

b $b = \sqrt{3+\sqrt{3}}$, gradient $-\dfrac{1}{2\sqrt{3+\sqrt{3}}}$.

17 a When $t = 20$ the volume is 30 cm^3.

b Pumping ceases when $t = 48$.

Exercise 10B PAGE 248

1 a 448 **b** 180 804

2 a 17 452 **b** 2 590 064

3 a 81 873 **b** 60 653

4 Approximately 680 grams.

5 Approximately 7.36 kg.

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6 ~1733 years

7 **a** 0.5 kg **b** 0.25 kg **c** 0.397 kg

8 98.6% 9 ~22%

10 Approximately 4200 years.

11 Approximately 117 years.

12 **b** Easier to divide into 72 mentally as it is an integer with many factors.

13 12.9 minutes, i.e. approximately 13 minutes. Compare your answer regarding the forensic possibilities of this idea with that of others in your class.

Exercise 10C PAGE 252

1 **a** 0.885

b Approximately 18.122 million.

2 Approximately 53 500.

3 Various ways of writing the answer, two of which are shown below.

$$y = \frac{150e^{0.6x}}{1+0.5e^{0.6x}} = \frac{300}{1+2e^{-0.6x}}$$

4 **a** According to the model, the limiting value of L is 200.

This means that when fully grown the length of the animal is 2 metres (or as near as makes no difference.)

b Various ways of writing the answer, three of which are shown below.

$$L = \frac{10\,200e^{0.4t}}{149+51e^{0.4t}} = \frac{10\,200}{51+149e^{-0.4t}} \approx \frac{200}{1+2.9216e^{-0.4t}},$$

the last of these being in the $\dfrac{K}{1+Ce^{-at}}$ form.

c Approximately 189.8 cm

5 **a** $P = \dfrac{2500}{1+14.625e^{-0.2t}}$

b 2500

c Approximately 839

6 Approximately 17 175

Exercise 10D PAGE 255

1 $\dfrac{dy}{dx} = 1$, Graph F.

2 $\dfrac{dy}{dx} + 2 = 0$, Graph J.

3 $\dfrac{dy}{dx} = 4 - 2x$, Graph D.

4 $\dfrac{dy}{dx} = x(x-3)$, Graph I.

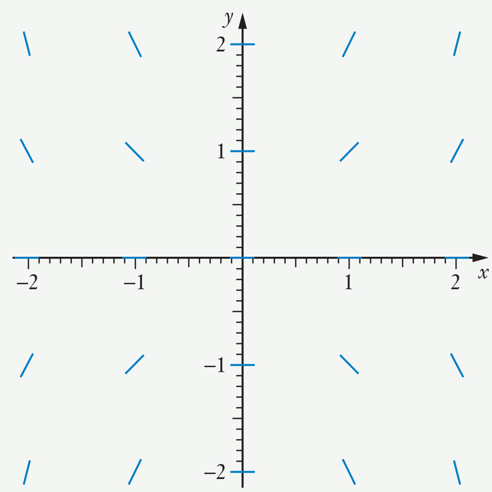
5 $\dfrac{dy}{dx} = (x+1)(3-x)$, Graph E.

6 $\dfrac{dy}{dx} = \sqrt{x}$, Graph G.

7 $\dfrac{dy}{dx} = 2^x$, Graph H.

8 $\dfrac{dy}{dx} = \dfrac{x}{2}$, Graph C.

9



Miscellaneous exercise ten PAGE 260

1 $\dfrac{dy}{dx} = y - x$, slope field B.

$\dfrac{dy}{dx} = \dfrac{x}{y}$, slope field C.

$\dfrac{dy}{dx} = y - 1$, slope field A.

3 $\delta y \approx 0.055$

4 **a** $2\cos x$ **b** $2\sin x\cos x$

c $\cos(\sin x).\cos x$ **d** $\dfrac{19}{(5-3x)^2}$

e $6(2x+3)^2$ **f** $\dfrac{3\cos x - 2y}{2x+3y^2}$

g $\dfrac{2x^2 - y}{x(\ln x - 6y)}$ **h** $\dfrac{(5-3y)(1+2y)}{3(2xy+x-2)}$

5 $2y^2 = e^{2x} + 17$

6 $y = -\dfrac{11}{3}x - \dfrac{10}{3}$ [at the point (1, −7)]

and $y = -\dfrac{4}{3}x + \dfrac{10}{3}$ [at the point (1, 2)].

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7 6π units^3

8 **a** $\dfrac{(3x^2-5)^8}{48}+c$

b $\dfrac{(8x+5)(x-5)^8}{72}+c$

c $8\sqrt{x^2-3}+c$

d $\dfrac{4}{75}(5x-2)^{\frac{3}{2}}(15x+4)+c$

e $-4\cos(x^2-5)+c$

f $\dfrac{(1+e^x)^5}{5}+c$

g $\dfrac{8}{3}\sqrt{x-3}(x+6)+c$

h $-\dfrac{5+4x}{2(x+2)^2}+c$

9 πr^2 cm^3/s **a** 100π cm^3/s **b** 16 cm

10 0.006 rad/s

11 $\dfrac{27}{16\pi}$ cm/s

13 **a** ~76.6 million tonnes **b** ~17.9 years

Exercise 11A PAGE 267

1 **a** 24 m/s^2 **b** 130 m

2 0.3 m/s

3 **a** –1 m/s **b** 0 m/s^2

4 **a** 4 m/s^2 **b** 7 m

5 **a** 4.5 m/s **b** $\left(5+\dfrac{4\pi}{3}-\dfrac{\sqrt{3}}{4}\right)$ m

6 12 m/s^2

7 8 m/s

8 **a** 0.5 **b** $0.1e^2$

9 **a** $\dfrac{1}{(2t+3)^2}$ m/s, $-\dfrac{4}{(2t+3)^3}$ m/s^2

b 0.4 m, 0.04 m/s, –0.032 m/s^2

10 7, 41 m/s

11 **a** 24 m/s **b** 8, 64 m

c 40 m **d** 13 m

12 –10 m/s

13 **a** 2 m/s **b** $(4\cos 2t)$ m/s^2

c 4 m/s^2 **d** $(1-\cos 2t)$ m

e 2 m

14 **a** $(9x+6)$ m/s^2 **b** 14 m/s, 42 m/s^2

15 0.5 ln (8.5) metres

Exercise 11B PAGE 275

1 **a** 5 m, π seconds **b** 4 m, 0.4π seconds

c 2 m, 0.5π seconds

2 **a** π seconds **b** 2π seconds

c 0.4π seconds

3 **a** $x=\sin 0.5t$ **b** $x=-\sin 0.5t$

c $x=3\sin 2t$ **d** $x=-0.5\sin \pi t$

4 **a** $x=2\cos 2t$ **b** $x=1.5\cos 4t$

c $x=0.5\cos 4\pi t$

5 **a** $x=\pm 2.5\sin 2t$ **b** 2.5 m/s

6 **a** $\sqrt{34}$ m, 0.4π s **b** $\sqrt{58}$ m, π s

7 **b** 20 seconds, 4 m **c** 2.35 m

8 **b** 6 seconds, 2 m **c** $(4-\sqrt{3})$ m

9 **b** π seconds, 3 m **c** 2.76 m

10 **a** $x=4\sin\left(\pi t+\dfrac{5\pi}{6}\right)$ **b** 4π m/s

11 **a** $x=2\sin\left(5t+\dfrac{\pi}{4}\right)$ **b** 10 m/s

c 50 m/s^2

12 **a** $\dfrac{3\sqrt{3}}{10}$ m **b** $\dfrac{3\sqrt{3}}{10}$ m

c **i** $\dfrac{\pi}{12}$ **ii** $\dfrac{5\pi}{12}$ **iii** $\dfrac{7\pi}{12}$

13 **a** $-\dfrac{3\sqrt{3}}{2}$ m **b** $-\dfrac{3\pi}{2}$ m/s

c $\dfrac{3\pi}{2}$ m/s **d** $\dfrac{2}{3}$

14 **a** 0.96 seconds

b 0.19 seconds

c 0.42 seconds

d 0.84 seconds, 2.30 seconds

15 0.72 seconds

16 **a** $x=2\sin 2t$ **b** $x=4\cos 2t$

17 **a** 2 cm **b** $\dfrac{\pi}{4}$ seconds

c $\dfrac{\pi}{16}$ seconds **d** 16 cm/s

e $\dfrac{\pi}{48}$ seconds

18 **a** 4 m **b** 2

c 14.98 m

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19 **b** 2 seconds, 4 m **c** 3 m
d 7 m

20 **b** π seconds, 3 m **c** 5 m
d 2 m

21 **a** 0.21 m **b** 0.27 m

22 π seconds, 25 m

23 $\frac{2\pi}{3}$ seconds, 0.65 m

Miscellaneous exercise eleven PAGE 279

1 **a** $6xy$ **b** $\dfrac{15-4y+8\cos 2x}{4x+5y^4}$

2 Approx 34.7 years

3 **a** 0 m/s **b** $(3\sin 2t)$ m/s^2

c $\frac{\pi}{4}$ **d** $(1.5t - 0.75\sin 2t)$ m

e $\dfrac{2\pi - 3\sqrt{3}}{8}$ m

4 **a** $6x(3x^2-2)$ m/s^2 **b** 1 m/s, 6 m/s^2

5 $y^3 = x^2 - 3$

6 0.04 rad/s, 1.6 m/s

7 128π units^3 **8** $\frac{2}{3}$ units^2

9 **a** $\frac{28}{3}$ units^2 **b** $\frac{824\pi}{15}$ units^3

c 24π units^3

10 10 m/s

11 **a** $\frac{\pi}{2}$ seconds, 0, 0 m **b** $\frac{2\pi}{3}$ seconds, 5, 0 m

c π seconds, 2, 0 m **d** $\frac{2\pi}{5}$ seconds, 1, 1 m

12 $c = 5, d = 6, k_1 = 2, k_2 = 0.5$ Time period for A is π seconds and for B is 4π seconds.

Exercise 12A PAGE 293

1 The sample means will be approximately normally distributed with a mean of 3.5 and a standard deviation of 0.24 (i.e. $\frac{1.71}{\sqrt{50}}$).

If instead a sample size of 150 were used the distribution would still be approximately normal with a mean of 3.5 but with a smaller standard deviation than before, now 0.14 (i.e. $\frac{1.71}{\sqrt{150}}$).

2 The sample means will be approximately normally distributed with a mean of 2.375 and a standard deviation of 0.09 (i.e. $\frac{0.696}{\sqrt{60}}$).

If instead a sample size of 100 were used the distribution would still be approximately normal with a mean of 2.375 but with a smaller standard deviation than before, now 0.07 (i.e. $\frac{0.696}{\sqrt{100}}$).

3 The 100 sample means will be approximately normally distributed with a mean of 7 and a standard deviation of 0.40 (i.e. $\frac{2.415}{\sqrt{36}}$).

If instead a sample size of 120 were involved the sample means would still be approximately normally distributed with mean of 7 but with a smaller standard deviation than before, now 0.22 (i.e. $\frac{2.415}{\sqrt{120}}$).

4 0.946 **5** 0.040 **6** 0.685

7 **a** Y will be normally distributed with mean 5 and standard deviation 0.2 i.e. $Y \sim N(5, 0.2^2)$.

b 0.006

8 $Y \sim N(30, 0.24)$, i.e. normally distributed with mean 30 and standard deviation $\sqrt{0.24}$.

9 **a** 0.4% **b** 25% **c** 0.402

10 **a** 0.006 **b** 0.202 **c** 0.938

11 We would expect the mean length of samples of ten adult male lizards of this species to be normally distributed with mean 17.4 and standard deviation $\frac{2.1}{\sqrt{10}}$ cm, i.e a standard deviation of approximately 0.664 cm. Thus a sample mean of 19.4 cm is just over three standard deviations above the mean. Whilst not impossible this is very unlikely. We would expect less than 0.13% of such samples to have a mean length this high. Hence, whilst it is possible that the sample of ten could be a 'freakish' sample we would be wise to consider other possible reasons for the surprising sample mean. Was the sample really a random sample? Perhaps the lizards were caught in a region where larger than normal lizards of this species were found. Perhaps the scientists' confidence in the assumption of a normal distribution or in the given population mean and standard deviation was misplaced. Were all of the lizards in the sample really adult males of this species? Etc.

12 **a** $\frac{1}{3}$ **b** $\sqrt{3}$ **c** 0.023

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13 **a** Sample means are normally distributed with mean 513, standard deviation $\frac{26}{8}$.

1.96 standard deviations either side of 513 gives interval of $506.63 \rightarrow 519.37$.

505 is not in this interval.

Significant difference at the 5% level.

b Sample means are normally distributed with mean 513, standard deviation $\frac{26}{10}$.

1.96 standard deviations either side of 513 gives interval of $507.90 \rightarrow 518.10$

510 is in this interval.

There is not a significant difference at the 5% level.

Exercise 12B PAGE 300

1 The 90% confidence interval has the smaller width. (If you want to be more confident of catching the population mean you need a bigger net.)

2 The 95% confidence interval has the smaller width.

3 The bigger size sample will give the narrower 95% confidence interval.

4 565 cm $\le \mu \le$ 581 cm

5 25.12 kg $\le \mu \le$ 27.16 kg

6 16.51 cm $\le \mu \le$ 17.89 cm

7 Note that we can assume that the sample mean is from a normal distribution of sample means because, though the sample is small, the population the sample is taken from is normally distributed. The 95% confidence interval is 73.73 cm $\le \mu \le$ 75.47 cm.

We can be 95% confident that the mean length of 12 month old baby girls lies between 73.73 cm and 75.47 cm (because 95% of the 95% confidence intervals constructed in this way will contain the population mean).

8 17.18 cm $\le \mu \le$ 18.42 cm

We can be 90% confident that the mean length of three month old seedlings of the particular plant type will lie between 17.18 cm and 18.42 cm (because 90% of the 90% confidence intervals constructed in this way will contain the population mean).

9 17.93 cm $\le \mu \le$ 18.67 cm, 17.99 cm $\le \mu \le$ 18.61 cm

Exercise 12C PAGE 303

1 110 **2** 372

3 23 (Okay to have a sample less than 30 as sample is taken from a normally distributed variable so sample means will be normally distributed.)

4 48 **5** 200 **6** 35

Miscellaneous exercise twelve PAGE 304

1 **a** 7 **b** $4 + 2x$

c $\frac{x^2}{4} - 24x + 800$ **d** $\frac{2}{\sqrt{x}} + \frac{1000}{x^2}$

2 $-\frac{x(3x+4y)}{2x^2+3y^2}$

3 **a** $\frac{x^3 - y}{x + y^3}$ **b** $\frac{7}{3}$

4 Slope field A, $y' = y$. Slope field B, $y' = x$. Slope field C, $y' = (x-3)(y-2)$.

5 $3x + \ln|x+1| + \ln|x^2 - 2| + c$

6 **a** $\sin^3 k$ units^2 **b** $(2 - \sin^3 k)$ units^2

7 **a** 0.32 m/s^2 **b** $50\sqrt{e} - 30$

8 **a** $3e^{2t} + 1$ **b** $3e + 1$ **c** 0.06

9 **a** 6.93 years **b** 13.86 years **c** 20.79 years

10 **a** 52.19 $\le$ population mean $\le$ 54.29

b 51.99 $\le$ population mean $\le$ 54.49

c 51.59 $\le$ population mean $\le$ 54.89

11 217.4 hours to 228.6 hours

We can be 95% confident that the mean life time of the population of triple A batteries of this brand will lie between 217.4 hours and 228.6 hours (because 95% of such 95% confidence intervals will contain the population mean).

12 8.40 a.m. to 6 p.m. **13** 0.044

14 0.4 m/s **15** $\frac{4}{41}$ rad/sec

16 **a** 0.0334 **b** 0.5889 **c** 0.3085

17 36 cm, $\frac{1}{2\pi}$ cm/s

18 About 5 minutes to 8 that morning. (The mathematics suggests 7.54 a.m.)

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19 **a** 62 (or more).

b If we use the standard deviation of the population as 65 we could be 95% confident that the population mean lies in the interval 1787 hours to 1813 hours (i.e. 1800 ± 13). The claimed mean of 1850 hours is well outside this range and casts very serious doubt about the legitimacy of the claimed mean value.

20 $2x + \ln|x+1| + 3\ln|x+2| - \ln|x-3| + c$

21 **a** $N = \dfrac{6250}{1 + 24e^{-0.4t}}$

b As $t \to \infty$, $e^{-0.4t} \to 0$ and so $N \to 6250$.

c **i** 1970 **ii** 5220

23 **a** $\sin^{-1} x + c$

b $\sin^{-1}\left(\dfrac{x}{5}\right) + c$

c $\dfrac{1}{2}\sin^{-1}\left(\dfrac{2x}{3}\right) + c$

d $\dfrac{1}{2}\sin^{-1} x + \dfrac{1}{2}x\sqrt{1-x^2} + c$

e $2\sin^{-1}\left(\dfrac{x}{2}\right) + \dfrac{1}{2}x\sqrt{4-x^2} + c$

f $\dfrac{x}{2}\sqrt{4-x^2} - 2\cos^{-1}\left(\dfrac{x}{2}\right) + c$

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